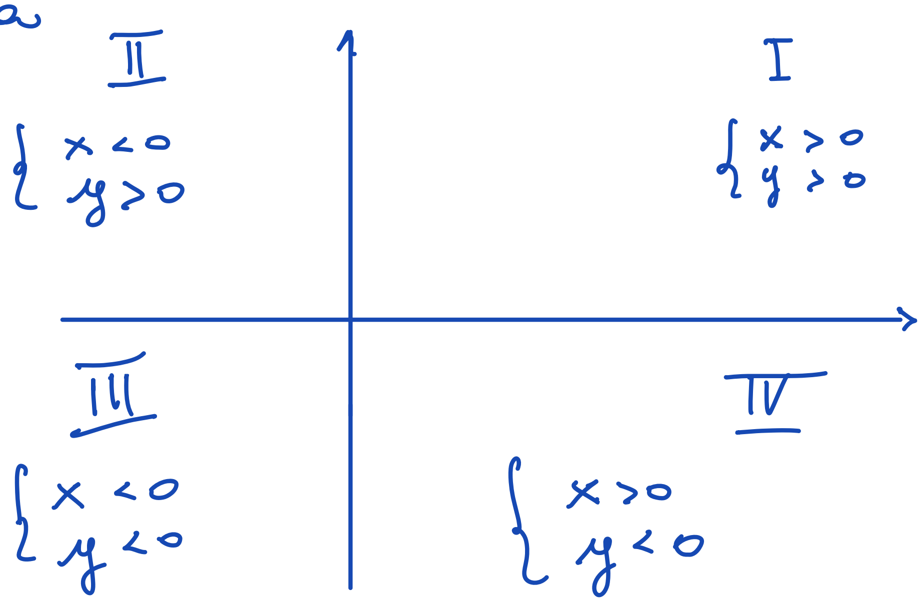
Geometria analitica

$P(x, y)$

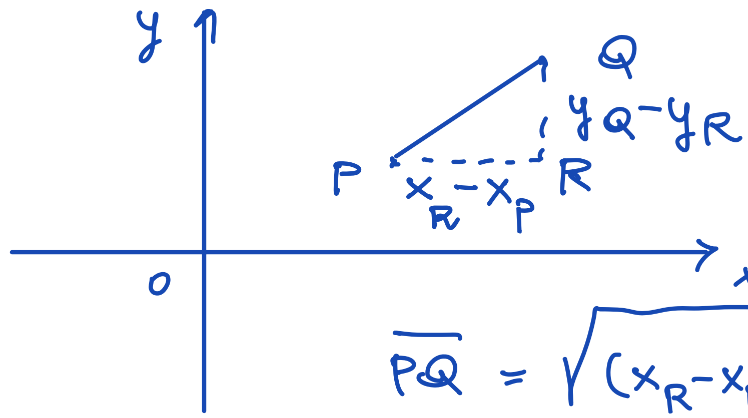


$$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 = \{ (x,y) \,/\, x, y \in \mathbb{R} \}$$

$$P(x,y), \quad Q(z,w) \in \mathbb{R}^2$$

$$d: \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow [0, +\infty[$$

$$d((x,y),(z,w)) = \sqrt{(x-z)^2 + (y-w)^2}$$



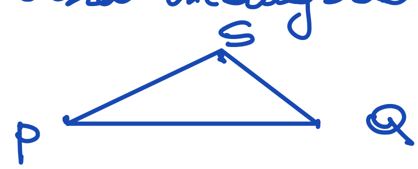
$$\overline{PQ} = \sqrt{(x_R - x_P)^2 + (y_Q - y_R)^2}$$

① $d(P,Q) \geq 0 \quad \forall\, P,Q$; $d(P,Q) = 0 \iff P \equiv Q$

② $d(P,Q) = d(Q,P) \quad \forall\, P,Q$

③ $d(P,Q) \leq d(P,S) + d(Q,S) \quad \forall\, P,Q,S$

disuguaglianza triangolare



Esercizio $P=(0,0)$ $Q=(3,0)$ $R=(3,4)$

1. Calcolare $d(P,Q)$, $d(P,R)$ e $d(Q,R)$
2. Verificare la disuguaglianza triangolare.

1. $d(P,Q) = |0-3| = 3$

   $d(P,R) = \sqrt{(0-3)^2+(0-4)^2} = 5$

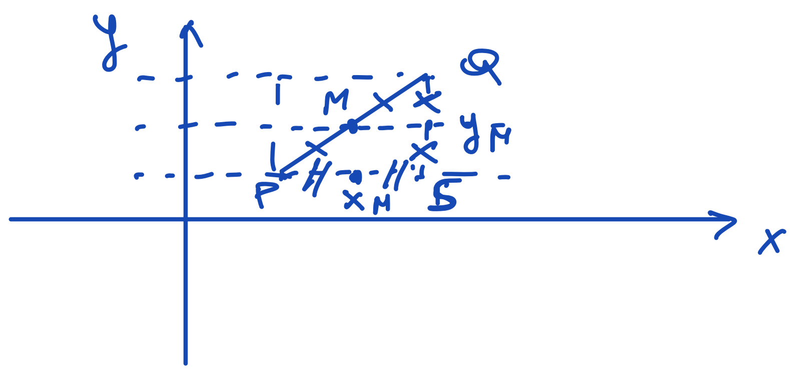
   $d(Q,R) = |0-4| = 4$

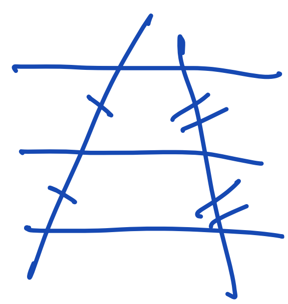
2. $d(P,Q) \leq d(P,R) + d(Q,R)$

   $3 \leq 5 + 4$ verificato

Punto medio di $P(x_P, y_P)$, $Q(x_Q, y_Q)$

$$M_{PQ}\left(\frac{x_P+x_Q}{2}; \frac{y_P+y_Q}{2}\right)$$





Equazione di una retta

$ax + by + c = 0$ eq. retta in forma implicita

$by = -ax - c$ $b \neq 0$

$y = -\frac{a}{b}x - \frac{c}{b}$ $m = -\frac{a}{b}$ pendenza

$q = -\frac{c}{b}$ intercetta

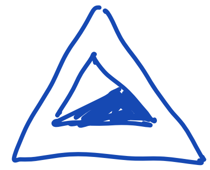
$b=0$ $by + c = 0$

eq. retta // asse y

$y = mx + q$ o $x = k$

$P_1(x_1, y_1)$ $P_2(x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ coefficiente angolare



10%

$\tan \alpha = \frac{1}{10}$

$\alpha = 5{,}71°$

$y = mx + q \vee x = k$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Esercizio Calcolare l'equazione della retta

1. passante per $P(-2;-1)$ e $Q(1,-3)$
2. " " $P(-1,1)$ e $Q(1;-2)$
3. " " $P(1,-2)$ e $Q(2,2)$
4. " " $P(-2,1)$ e $Q(2;1)$
5. " " $P(2,3)$ e $Q(2,5)$

1. $m_1 = \frac{-1+3}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$

$y = -\frac{2}{3}x + q$ $\quad -1 = -\frac{2}{3}(-2) + q$

$q = -\frac{4}{3} - 1$ $\quad q = -\frac{7}{3}$

$$\boxed{y = -\frac{2}{3}x - \frac{7}{3}}$$

$-1 = \frac{4}{3} - \frac{7}{3}$

$-3 = -\frac{2}{3} - \frac{7}{3}$ Vero

(2) $(-1,1) \quad (1,-2)$

$$m = \frac{1+2}{-1-1} = -\frac{3}{2}$$

$$y = mx + q$$

$$1 = \frac{3}{2} + q$$

$$q = -\frac{1}{2}$$

$$\boxed{y = -\frac{3}{2}x - \frac{1}{2}}$$

$1 = \frac{3}{2} - \frac{1}{2}$ Sì $\qquad -2 = -\frac{3}{2} - \frac{1}{2}$ Sì

(3) $(1,-2) \quad (2,2)$

$$m = \frac{-2-2}{1-2} = \frac{-4}{-1} = 4$$

$$y = mx + q$$

$$y = 4x + q$$

$$-2 = 4 + q$$

$$q = -6$$

$$\boxed{y = 4x - 6}$$

$-2 = 4 - 6$ Sì

$2 = 8 - 6$ Sì

(4) $(-2,1) \quad (2,1) \qquad y = 1$

(5) $(2,3) \quad (2,5) \qquad x = 2$

$y - y_0 = m(x - x_0)$ eq fascio proprio di rette

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \qquad x_1 \neq x_2 \quad y_1 \neq y_2$$

eq. retta passante per due punti.

$$P(t) = (x(t), y(t)) = t \cdot P + (1-t)Q$$

$$(x(t), y(t)) = (t x_1, t y_1) + ((1-t)x_2, (1-t)y_2)$$

$$\begin{cases} x(t) = t x_1 + (1-t)x_2 \\ y(t) = t y_1 + (1-t) y_2 \end{cases} \qquad \begin{cases} x(t) = t x_1 + x_2 - t x_2 \\ y(t) = t y_1 + y_2 - t y_2 \end{cases}$$

$$t = \frac{x(t) - x_2}{x_1 - x_2} \qquad \begin{cases} x(t) - x_2 = t(x_1 - x_2) \\ y(t) - y_2 = t(y_1 - y_2) \end{cases}$$

$$t = \frac{y(t) - y_2}{y_1 - y_2}$$

Esempio

$$\begin{cases} x = 2t + 1 \\ y = 3t \end{cases} \qquad P(2t+1, 3t) \quad t \in \mathbb{R}$$

eq. retta in forma parametrica

$$\begin{cases} t = \frac{y}{3} \\ x = \frac{2}{3}y + 1 \end{cases} \Rightarrow 3x = 2y + 3$$

$$2y = 3x - 3 \qquad y = \frac{3}{2}x - \frac{3}{2}$$

Esercizio: Calcolare l'equazione della retta passante per $P(1;2)$ e $Q(2;3)$

$$m = \frac{2-3}{1-2} = \frac{-1}{-1} = 1$$

$$y - 2 = 1(x-1)$$

$$y = x + 1$$

$$\begin{cases} x = t \cdot 1 + (1-t)\,2 \\ y = t \cdot 2 + (1-t)\,3 \end{cases} \qquad \begin{cases} x = t + 2 - 2t \\ y = 2t + 3 - 3t \end{cases}$$

$$\begin{cases} x = 2 - t \\ y = 3 - t \end{cases} \qquad P\,(2-t\,,\,3-t)$$

Equazione del fascio proprio $y - y_0 = m\,(x - x_0)$

Equazione del fascio improprio di rette

$$y = \overline{m}\,x + q$$

Es: $y = -\frac{1}{3}x + 9$ $\qquad \overline{m} = -\frac{1}{3}$

$r_1 \parallel r_2 \iff m_1 = m_2$ Condizione di parallelismo fra rette

$r_1: y = m_1 x + q_1 \qquad r_2: y = m_2 x + q_2$

Esercizio: eq. retta: $(1;10)$ e $\parallel y = 2x + 5$

$$y - y_0 = m\,(x - x_0)$$

$$y - 10 = 2\,(x - 1) \qquad y = 2x + 8$$

$$q = 8$$

$r_1 \perp r_2 \iff m_1 \cdot m_2 = -1$ Condizione di perpendicolarità fra rette

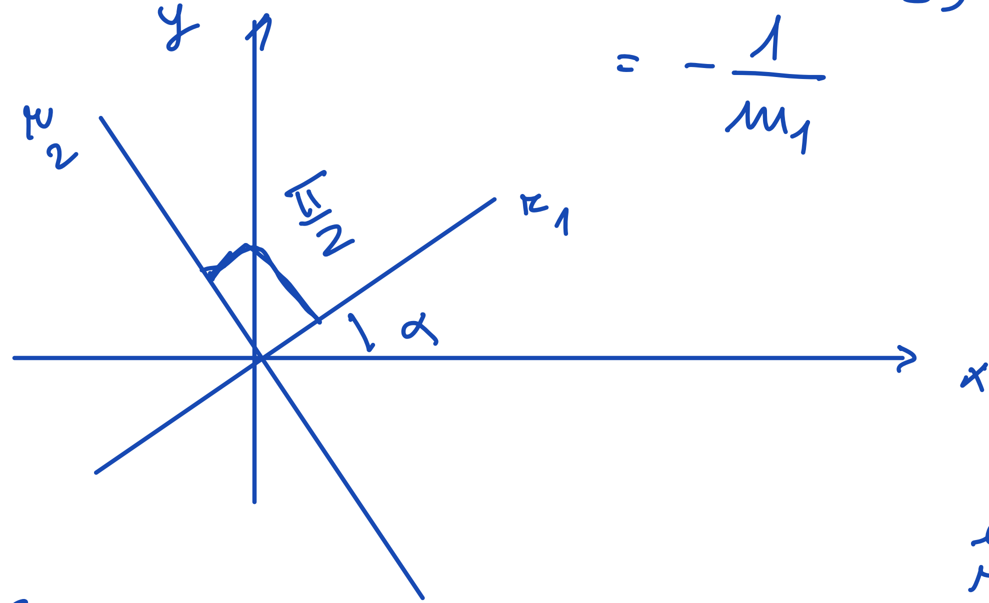
$$m_1 = -\frac{1}{m_2}$$

$r_1: y = m_1 x + q_1 \qquad r_2: y = m_2 x + q_2$

$m_1 = \tan\alpha \qquad m_2 = \tan\left(\frac{\pi}{2} + \alpha\right)$

$$m_2 = \tan\left(\alpha + \frac{\pi}{2}\right) = \frac{\sin\left(\alpha + \frac{\pi}{2}\right)}{\cos\left(\alpha + \frac{\pi}{2}\right)} = \frac{\cos\alpha}{-\sin\alpha} = -\frac{1}{\tan\alpha} =$$

$$= -\frac{1}{m_1}$$



Esercizio : $r: y = 2x$ eq. retta passante per $P(2,5)$ e $\perp r$

$$m_\perp = -\frac{1}{2}$$

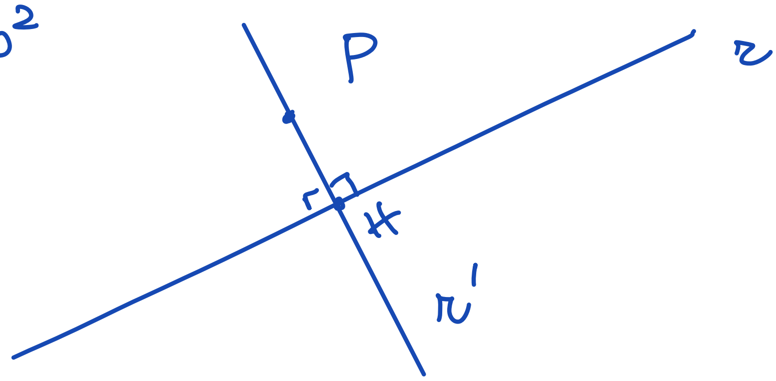
$$y - 5 = -\frac{1}{2}(x-2) \qquad y - 5 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 6$$

Distanza di un punto da una retta $\quad P(x_0, y_0)$

$r: ax + by + c = 0$

$$d(P, r) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$



$r: \; y = mx + q \qquad P(x_0, y_0)$

$$d(P, r) = \frac{|y_0 - m x_0 - q|}{\sqrt{1+m^2}}$$

Esercizio: $\quad r: \; y = 3x - 5 \qquad P \equiv (1; 2)$

$$d(P, r) = \frac{|2 - 3 \cdot 1 + 5|}{\sqrt{1+9}} = \frac{4}{\sqrt{10}} = \frac{\cancel{4}^{\,2}\sqrt{10}}{\cancel{10}_{\,5}}$$

$$3x - y - 5 = 0$$

$$d(P, r) = \frac{|3 \cdot 1 - 2 - 5|}{\sqrt{9+1}} = \frac{|-4|}{\sqrt{10}} = \frac{4}{\sqrt{10}} = \frac{\cancel{4}^{\,2}\sqrt{10}}{\cancel{10}_{\,5}}$$