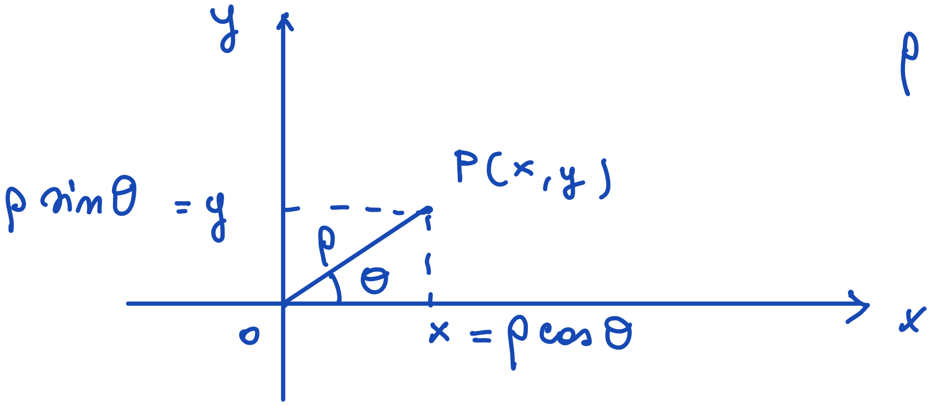
# Le coordinate polari




$\rho = r$

$\rho = d(P, O)$

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

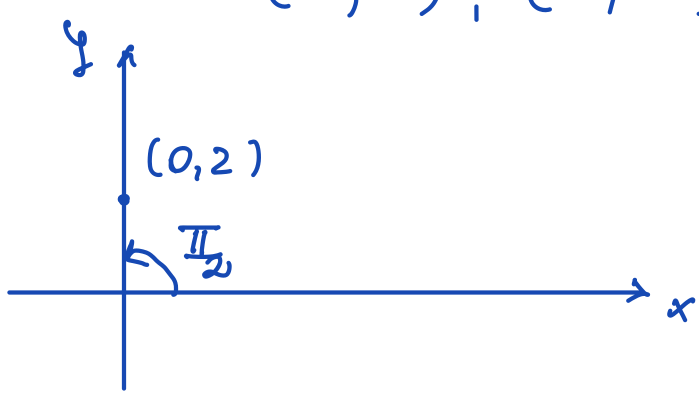
$P(x, y)$ $\qquad \rho = \sqrt{x^2 + y^2}$

$$\theta = \begin{cases} \arctan\frac{y}{x} & x > 0 \text{ e } y \geq 0 \\ \frac{\pi}{2} & x = 0 \text{ e } y > 0 \\ \frac{3}{2}\pi & x = 0 \text{ e } y < 0 \\ 2\pi + \arctan\frac{y}{x} & x > 0 \text{ e } y < 0 \end{cases}$$

**Esercizio** Determinate le coordinate polari dei punti che hanno coordinate cartesiane

$(0; 2)$, $(1; -1)$, $(-7\sqrt{3}; 7)$, $(-5; 0)$

1

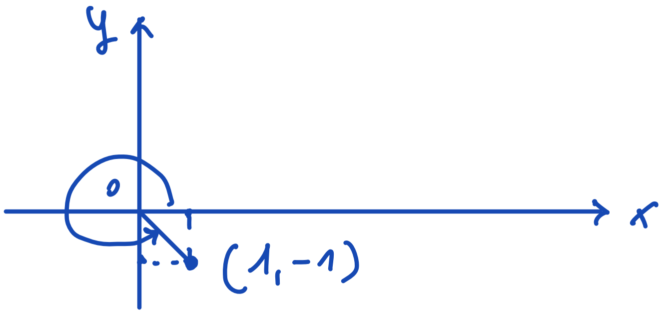



$\rho = \sqrt{0 + 2^2} = 2$

$\theta = \frac{\pi}{2}$

$(0, 2) \longrightarrow \left(2, \frac{\pi}{2}\right)$

2



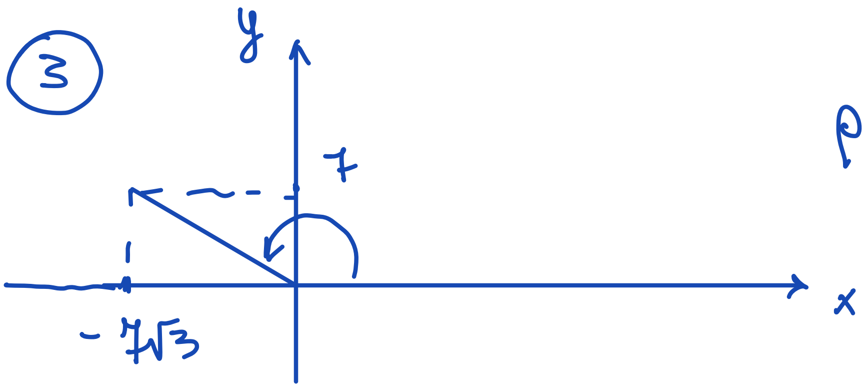

$$\rho = \sqrt{1+(-1)^2} = \sqrt{2}$$

$$\theta = 2\pi + \arctan\frac{-1}{1} =$$

$$= 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi$$

$$(1,-1) \longrightarrow \left(\sqrt{2}\,;\ \frac{7}{4}\pi\right)$$

3



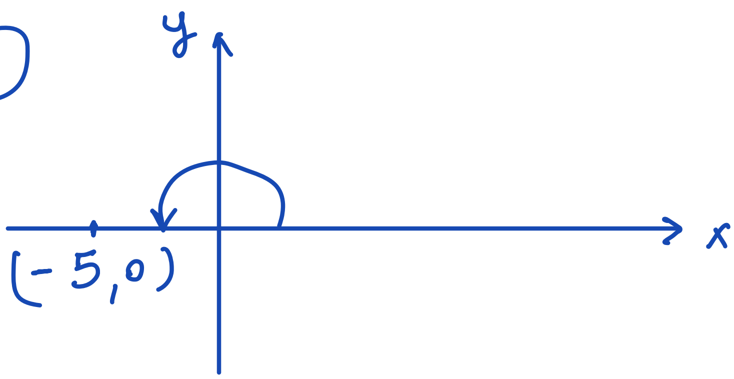

$$\rho = \sqrt{(-7\sqrt{3})^2+7^2} = 7\sqrt{3+1} = 14$$

$$\theta = \pi + \arctan\frac{\cancel{7}^{\,1}}{-\cancel{7}\sqrt{3}} =$$

$$= \pi - \frac{\pi}{6} = \frac{5}{6}\pi$$

$$(-7\sqrt{3}, 7) \longrightarrow \left(14\,;\ \frac{5}{6}\pi\right)$$

4




$$\rho = 5$$

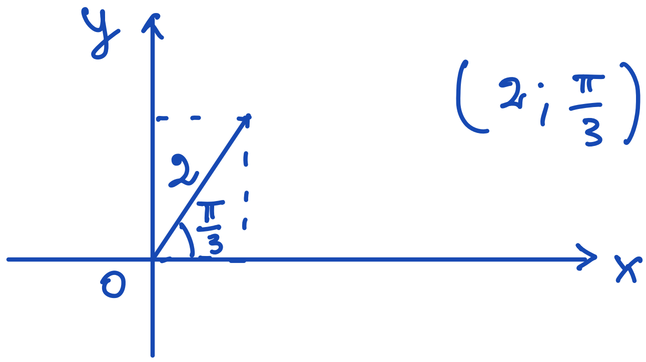
$$\theta = \pi + \arctan\frac{0}{-5} = \pi$$

$$(-5,0) \longrightarrow (5,\pi)$$

Esercizio:

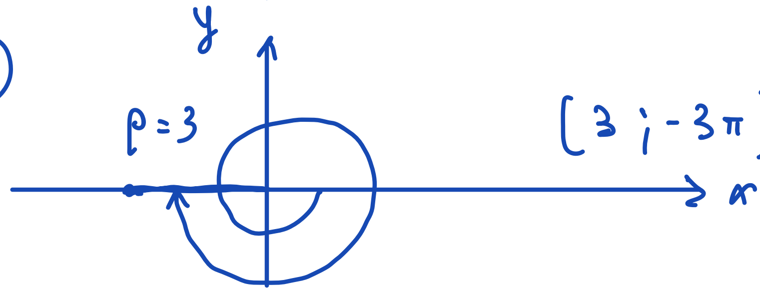
Determinate le coordinate cartesiane dei punti che hanno coordinate polari $(\rho, \theta)$ uguali a $\left(2; \frac{\pi}{3}\right)$, $(3; -3\pi)$, $\left(1; \frac{5}{4}\pi\right)$, $\left(6; \frac{23}{6}\pi\right)$

1



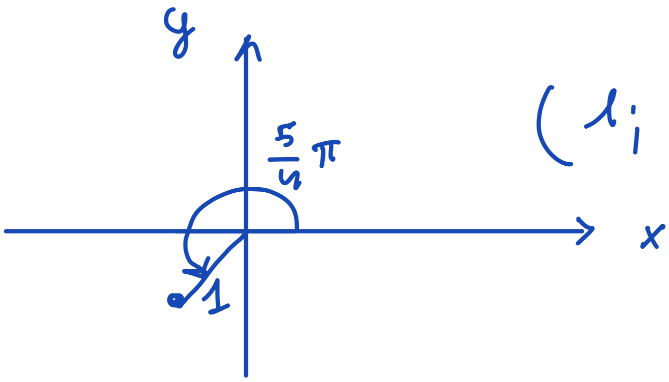

$$\left(2;\frac{\pi}{3}\right)\to\left(2\cos\frac{\pi}{3};\,2\sin\frac{\pi}{3}\right)=$$
$$=\left(2\,\frac{1}{2};\,2\cdot\frac{\sqrt{3}}{2}\right)=\left(1;\sqrt{3}\right)$$

2



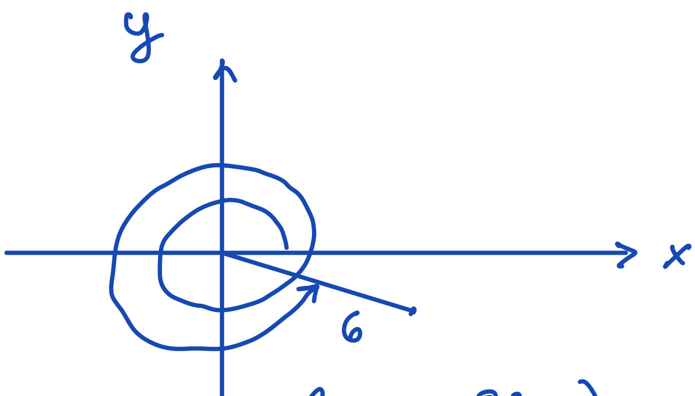

$$\left(3;-3\pi\right)\to\left(3\cos(-3\pi),\,3\sin(-3\pi)\right)=$$
$$=\left(3\cdot(-1),\,3\cdot 0\right)=\left(-3,0\right)$$

3




$$\left(1;\frac{5}{4}\pi\right)\to\left(1\cdot\cos\frac{5}{4}\pi,\,1\cdot\sin\frac{5}{4}\pi\right)=$$
$$=\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$$

4




$$\frac{23}{6}\pi=\frac{24-1}{6}\pi=$$
$$=4\pi-\frac{\pi}{6}$$

$$\left(6;\frac{23}{6}\pi\right)\to\left(6\cos\frac{23}{6}\pi,\,6\sin\frac{23}{6}\pi\right)=$$
$$=\left(6\cos\left(-\frac{\pi}{6}\right),\,6\sin\left(-\frac{\pi}{6}\right)\right)=\left(6\cdot\frac{\sqrt{3}}{2},\,6\left(-\frac{1}{2}\right)\right)$$
$$=\left(3\sqrt{3},\,-3\right)$$

Esercizio: trovate la legge per ottenere seno e coseno degli angoli $-x$, $x+\pi$, $\pi-x$ e $\frac{\pi}{2}-x$ conoscendo seno e coseno di x.

$\sin(-x) = -\sin x$ poiché $\sin x$ è una funzione dispari.

$\cos(-x) = \cos x$ poiché $\cos x$ è una funzione pari

$\sin(x+\pi) = \sin x \cos\pi + \sin\pi \cos x = -\sin x$

$\cos(x+\pi) = \cos x \cos\pi - \sin x \sin\pi = -\cos x$

$\sin\left(\frac{\pi}{2}-x\right) = \sin\frac{\pi}{2}\cos x - \sin x \cos\frac{\pi}{2} = 1\cdot\cos x = \cos x$

$\cos\left(\frac{\pi}{2}-x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x = \sin x$

Esercizio: determinate seno, coseno e tangente degli angoli di ampiezza $-\frac{\pi}{6}$, $\frac{3}{4}\pi$, $\frac{\pi}{4}+\frac{\pi}{3}$, $\frac{\pi}{3}-\frac{\pi}{4}$, $\frac{\pi}{8}$.

$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\left(-\frac{\pi}{6}\right) = \left(-\frac{1}{2}\right) : \frac{\sqrt{3}}{2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\sin\left(\frac{3}{4}\pi\right) = \sin\left(\pi-\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{3}{4}\pi\right) = \cos\left(\pi-\frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\left(\frac{3}{4}\pi\right) = \frac{\sqrt{2}}{2} : \left(-\frac{\sqrt{2}}{2}\right) = -1$

$\sin\left(\frac{7}{3}\pi\right) = \sin\left(2\pi+\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{7}{3}\pi\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$$\tan\left(\frac{7}{3}\pi\right) = \frac{\sqrt{3}}{2} : \left(\frac{1}{2}\right) = \sqrt{3}$$

$$\frac{7}{3}\pi = \frac{6\pi+\pi}{3} = 2\pi + \frac{\pi}{3}$$

$$\frac{17}{7}\pi = \frac{14+3}{7}\pi = 2\pi + \frac{3}{7}\pi$$

$$\frac{27}{4}\pi = \frac{24+3}{4}\pi = 6\pi + \frac{3}{4}\pi$$

$$\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} =$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$\tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \frac{\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}} = \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{(1+\sqrt{3})^2}{1-3} =$$

$$= \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$$

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{12} = \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} =$$

$$= \sqrt{\frac{4-2\sqrt{3}}{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{12}=\sqrt{\frac{1+\cos\frac{\pi}{6}}{2}}=\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}=$$

$$=\sqrt{\frac{2+\sqrt{3}}{4}}=\sqrt{\frac{4+2\sqrt{3}}{8}}=\frac{\sqrt{3}+1}{2\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}=\frac{1}{2}\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$$

$$\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\frac{(\sqrt{6}-\sqrt{2})^2}{6-2}=\frac{6+2-2\sqrt{12}}{4}=$$

$$=\frac{8-4\sqrt{3}}{4}=2-\sqrt{3}$$

$$\sin\frac{\pi}{8}=\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\sqrt{\frac{2-\sqrt{2}}{4}}=$$

$$=\frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\cos\frac{\pi}{8}=\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}=\sqrt{\frac{2+\sqrt{2}}{4}}=$$

$$=\frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\tan\frac{\pi}{8}=\frac{\sqrt{2-\sqrt{2}}}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{2+\sqrt{2}}}=\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$$