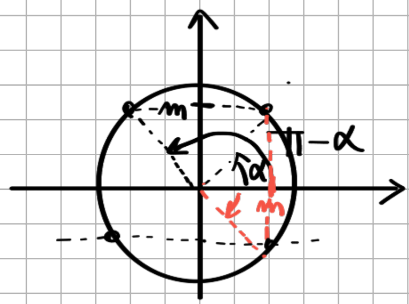
EQUAZIONI GONIOMETRICHE

1) ELEMENTARI

$\alpha = \arcsin(m)$



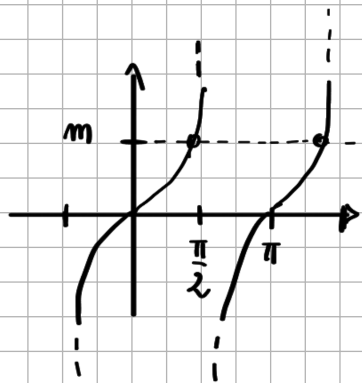
- $\sin x = m$ con $-1 \leq m \leq 1$

  $x = \arcsin(m) + 2k\pi,\ k \in \mathbb{Z}$

  $\vee\ x = \pi - \arcsin(m) + 2k\pi,\ k \in \mathbb{Z}$

- $\cos x = m$ $\quad -1 \leq m \leq 1$

  $x = \pm\arccos(m) + 2k\pi,\ k \in \mathbb{Z}$



- $\tan x = m,\ m \in \mathbb{R}$

  $x = \arctan(m) + k\pi,\ k \in \mathbb{Z}$

ES. (a) $\sin x = \frac{\sqrt{3}}{2}$

(b) $\cos x = \frac{1}{2}$

(c) $|\sin x| = \frac{1}{2}$

(a) $x = \frac{\pi}{3} + 2k\pi \quad \vee \quad x = \pi - \frac{\pi}{3} + 2k\pi =$

$= \frac{2}{3}\pi + 2k\pi$

(b) $x = \pm\frac{\pi}{3} + 2k\pi \rightarrow x = \frac{\pi}{3} + 2k\pi \quad \vee$

$x = 2\pi - \frac{\pi}{3} + 2k\pi = \frac{5}{3}\pi + 2k\pi$

(c) $\sin x = \frac{1}{2} \quad \vee \quad \sin x = -\frac{1}{2}$

$\Leftrightarrow \quad x = \frac{\pi}{6} + 2k\pi \quad \vee \quad x = \frac{5}{6}\pi + 2k\pi$

$\vee\ x = \pi + \frac{\pi}{6} + 2k\pi \quad \vee \quad x = -\frac{\pi}{6} + 2k\pi$

$= \frac{7}{6}\pi + 2k\pi \qquad = 2\pi - \frac{\pi}{6} + 2k\pi$

$= \frac{11}{6}\pi + 2k\pi$

2) $\circ\ \sin f(x) = \sin g(x)$

$$\iff f(x) = g(x) + 2k\pi \ \vee \ f(x) = \pi - g(x) + 2k\pi \ , \ k \in \mathbb{Z}$$

$\circ\ \cos f(x) = \cos g(x)$

$$\iff f(x) = \pm g(x) + 2k\pi \ , \ k \in \mathbb{Z}$$

$\circ\ \tan f(x) = \tan g(x)$

$$\iff f(x) = g(x) + k\pi \ , \ k \in \mathbb{Z}$$

ES.

(a) $\tan(3x) = \tan\left(x + \frac{\pi}{2}\right)$

(b) $\cos\left(\frac{\pi}{2} - x\right) = \cos\left(x - \frac{\pi}{3}\right)$

(c) $\sin x = \sin\left(2x - \frac{\pi}{2}\right)$

(a) $3x = x + \frac{\pi}{2} + k\pi \iff 2x = \frac{\pi}{2} + k\pi$

$$\iff x = \frac{1}{2}\left(\frac{\pi}{2} + k\pi\right) = \frac{\pi}{4} + k\frac{\pi}{2} \ , \ k \in \mathbb{Z}$$

(b) $\frac{\pi}{2} - x = x - \frac{\pi}{3} + 2k\pi \ \vee \ \frac{\pi}{2} \cancel{-x} = \cancel{-x} + \frac{\pi}{3} + 2k\pi$

$$\iff 2x = \frac{\pi}{2} + \frac{\pi}{3} + 2k\pi \ \vee \ \text{IMPOSSIBILE.}$$

$$\iff 2x = \frac{5}{6}\pi + 2k\pi \iff x = \frac{5}{12}\pi + k\pi \ , \ k \in \mathbb{Z}$$

(c) $x = 2x - \frac{\pi}{2} + 2k\pi \ \vee \ x = \pi - 2x + \frac{\pi}{2} + 2k\pi$

$$\iff x = \frac{\pi}{2} + 2k\pi \ \vee \ 3x = \frac{3\pi}{2} + 2k\pi$$

$$\iff \text{''} \ \vee \ x = \frac{\pi}{2} + \frac{2}{3}k\pi \ , \ k \in \mathbb{Z}$$

3) LINEARI IN SENO e COSENO

$$a\sin x + b\cos x + c = 0 \qquad a, b, c \in \mathbb{R}$$

- $c = 0 \rightarrow$ EQ. OMOGENEA $\quad a \sin x + b \cos x = 0$

DIVIDO ENTRAMBI I MEMBRI PER $\cos x$, $\cos x \neq 0$.

$$\Rightarrow a \tan x + b = 0 \Rightarrow \tan x = -\frac{b}{a} \Rightarrow 1)$$

$$\cos x = 0 \iff x = \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$$

DIRE SE È SOL.

ES. $\quad 3 \sin x + \sqrt{3} \cos x = 0$

$$\underset{\cos x \neq 0}{\iff} 3 \tan x + \sqrt{3} = 0 \iff \tan x = -\frac{\sqrt{3}}{3}$$

$$\iff x = \frac{5}{6}\pi + k\pi, \quad k \in \mathbb{Z} \quad \leftarrow \text{SOL.}$$

$x = \frac{\pi}{2} + k\pi$ È SOL?

$$3 \cdot (\pm 1) + \sqrt{3} \cdot 0 \overset{?}{=} 0 \qquad \underline{\text{NO}}$$

- $c \neq 0 \qquad a \sin x + b \cos x + c = 0$

→ FORMULE PARAMETRICHE

→ SI PONE $\sin x = Y$ E $\cos x = X$

$$\begin{cases} aY + bX + c = 0 \\ X^2 + Y^2 = 1 \end{cases}$$ E RISOLVO IL SISTEMA

ES. $\quad \sin\left(\frac{5}{6}\pi - x\right) - \cos\left(\frac{4}{3}\pi - x\right) = 2$

$$\iff \cos x + \sqrt{3} \sin x = 2$$

$\sin x = Y, \quad \cos x = X$

$$\begin{cases} X + \sqrt{3}\, Y = 2 \\ X^2 + Y^2 = 1 \end{cases} \iff \begin{cases} X = 2 - \sqrt{3}\, Y \\ (2 - \sqrt{3}\, Y)^2 + Y^2 = 1 \end{cases}$$

$$\iff 4 + 3Y^2 - 4\sqrt{3}\, Y + Y^2 = 1$$

$$\iff 4Y^2 - 4\sqrt{3}\, Y + 3 = 0$$

$$\Delta = 16 \cdot 3 - 4 \cdot 4 \cdot 3 = 0$$

$$Y = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$$

$$\begin{cases} \sin x = \frac{\sqrt{3}}{2} \\ \cos x = 2 - \sqrt{3} \cdot \frac{\sqrt{3}}{2} \end{cases} \iff \begin{cases} \sin x = \frac{\sqrt{3}}{2} \\ \cos x = \frac{1}{2} \end{cases}$$

$$= 2 - \frac{3}{2} \qquad \iff x = \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$$

4) EQUAZIONI DI 2° GRADO IN SENO E COSENO

$$a \sin^2 x + b \sin x \cos x + c \cos^2 x + d = 0$$

$$a, b, c, d \in \mathbb{R}$$

- $d = 0$

DIVIDO ENTRAMBI PER $\cos^2 x$, $\cos^2 x \neq 0$

$$a \tan^2 x + b \tan x + c = 0$$

$z := \tan x$

$$a \cdot z^2 + bz + c = 0 \qquad \text{EQ DI } 2^\circ \text{ GRADO}$$

ES. $6 \sin^2 x - \sqrt{3} \sin x \cos x - \cos^2 x = 0$

$\cos^2 x \neq 0$

$6 \tan^2 x - \sqrt{3} \tan x - 1 = 0$

$z := \tan x$

$6z^2 - \sqrt{3}z - 1 = 0$

$\Delta = 3 - 4 \cdot 6 \cdot (-1) = 27$

$$z_{1,2} = \frac{\sqrt{3} \pm \sqrt{27}}{12} = \frac{\sqrt{3} \pm 3\sqrt{3}}{12} = \begin{cases} \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3} \\ -\frac{2\sqrt{3}}{12} = -\frac{\sqrt{3}}{6} \end{cases}$$

$$\tan x = \frac{\sqrt{3}}{3} \ \vee \ \tan x = -\frac{\sqrt{3}}{6}$$

SOL. $\rightarrow$ $x = \arctan \frac{\sqrt{3}}{3} + k\pi \ \vee \ x = \arctan\left(-\frac{\sqrt{3}}{6}\right) + k\pi,$ $k \in \mathbb{Z}$

$$= \frac{\pi}{6} + k\pi$$

CONTROLLO SE $x = \frac{\pi}{2} + k\pi$ E' SOL:

$6 \cdot 1 - \sqrt{3} \cdot 0 - 0 \overset{?}{=} 0$ <u>NO</u>

- $d \neq 0$

Si riscrive d come

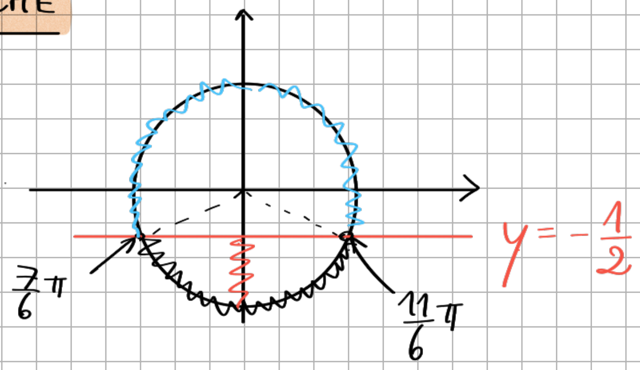
$$d \cdot 1 = d\left(\sin^2 x + \cos^2 x\right)$$

e mi riconduco al caso precedente.

ES. $2\cos^2 x + 3\sin^2 x + \sin x \cos x \underbrace{-3} = 0$

Disequazioni goniometriche

ES. $\sin x \leq -\frac{1}{2}$



$\sin x = -\frac{1}{2}$

$\iff x = \frac{7}{6}\pi + 2k\pi$

$\vee$

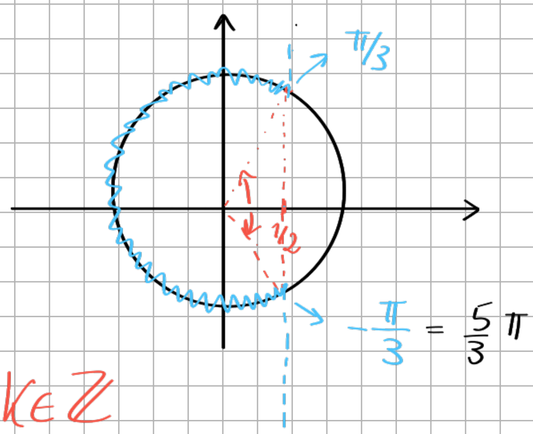
$x = \frac{11}{6}\pi + 2k\pi$

$\sin x \leq -\frac{1}{2} \iff \frac{7}{6}\pi + 2k\pi \leq x \leq \frac{11}{6}\pi + 2k\pi, \quad k \in \mathbb{Z}$

ES. $\sin x \geq -\frac{1}{2} \iff \cancel{\frac{11}{6}\pi + 2k\pi} \leq x \leq \frac{7}{6}\pi + 2k\pi$

$-\frac{\pi}{6} + 2k\pi \leq x \leq \frac{7}{6}\pi + 2k\pi, \quad k \in \mathbb{Z}$

ES. $\cos x \leq \frac{1}{2}$



$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} + 2k\pi$

$\vee$

$x = -\frac{\pi}{3} + 2k\pi \quad , k \in \mathbb{Z}$

$\frac{\pi}{3} + 2k\pi \leq x \leq \cancel{-\frac{\pi}{3} + 2k\pi} \quad , k \in \mathbb{Z}$

$2\pi - \frac{\pi}{3} + 2k\pi = \frac{5\pi}{3} + 2k\pi$

ES. $2\sin^2 x + \sin x - 1 < 0$

$t := \sin x$

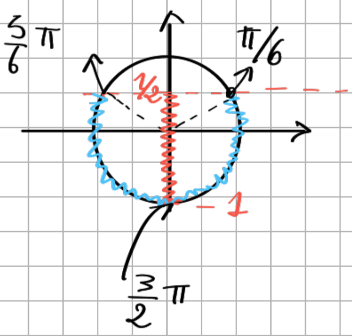
$2t^2 + t - 1 < 0$

$2t^2 + t - 1 = 0$

$t_{1/2} = \left\langle \begin{matrix} -1 \\ 1/2 \end{matrix} \right.$

$-1 < t < 1/2$

$-1 < \sin x < 1/2$



$\frac{5}{6}\pi + 2k\pi < x < \cancel{\frac{\pi}{6} + 2k\pi}$

$2\pi + \frac{\pi}{6} + 2k\pi$

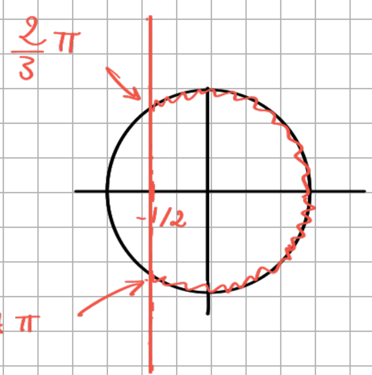
$= \frac{13}{6}\pi + 2k\pi$

ESCLUDO DALL' INSIEME SOL. $x = \frac{3}{2}\pi$.

$x \in \left(\frac{5}{6}\pi + 2k\pi, \frac{13}{6}\pi + 2k\pi\right) \setminus \left\{\frac{3}{2}\pi + 2k\pi\right\}, \quad k \in \mathbb{Z}$

ES. $\cos(\overbrace{x - \frac{\pi}{6}}^{y}) \geq -\frac{1}{2}$

$\rightarrow \cos y \geq -\frac{1}{2}$



$y \in \left[\frac{4}{3}\pi + 2k\pi, \frac{2}{3}\pi + 2\pi + 2k\pi\right]$

$y \in \left[\frac{4}{3}\pi + 2k\pi, \frac{8}{3}\pi + 2k\pi\right], \quad k \in \mathbb{Z}$

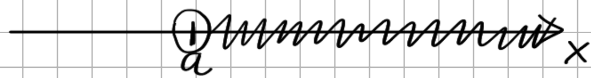
$\frac{4}{3}\pi + 2k\pi \overset{+\pi/6}{\leq} x - \cancel{\frac{\pi}{6}} \overset{+\pi/6}{\leq} \frac{8}{3}\pi + 2k\pi + \frac{\pi}{6}$

$\frac{\cancel{9}^{\,3}}{\cancel{6}_{\,2}}\pi + 2k\pi \leq x \leq \frac{17}{6}\pi + 2k\pi, \quad k \in \mathbb{Z}$

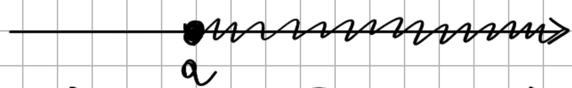
DISEQ. DI 1° GRADO

$x-a>0$ $(\geq,<,\leq)$, $a\in\mathbb{R}$

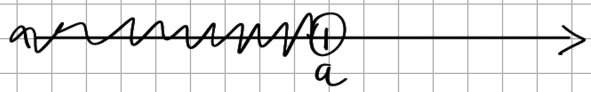
$$\{x\in\mathbb{R} \mid x-a>0\} = \{x\in\mathbb{R} \mid x>a\} = (a,+\infty)$$



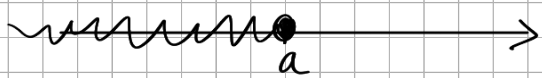
$$\{x\in\mathbb{R} \mid x-a\geq 0\} = \{x\in\mathbb{R} \mid x\geq a\} = [a,+\infty)$$



$$\{x\in\mathbb{R} \mid x-a<0\} = \{x\in\mathbb{R} \mid x<a\} = (-\infty,a)$$



$$\{x\in\mathbb{R} \mid x-a\leq 0\} = \{x\in\mathbb{R} \mid x\leq a\} = (-\infty,a]$$



DISEQ. DI 2° GRADO

$ax^2+bx+c>0$ $(\geq,<,\leq)$ $a,b,c\in\mathbb{R}$

SI PUO' SUPPORRE $a>0$: SE $a<0$ POSSO MOLTIPLICARE PER (-1) ENTRAMBI I MEMBRI ($\triangle$! VERSO DELLA DISUG.).

SE $a=0 \Rightarrow$ DISEQ. DI 1° GRADO

$a>0 \Rightarrow$ LA DISEQ. $ax^2+bx+c>0$ HA LE STESSE SOLUZIONI DI $x^2+\frac{b}{a}x+\frac{c}{a}>0$. QUINDI CI SI RICONDUCE AD UNA DISEQ. DEL TIPO

$$x^2+bx+c>0$$

3 CASI:

1) L'EQ. ASSOCIATA HA RADICI REALI DISTINTE

$x^2+bx+c=0$ HA 2 RADICI REALI $x_1<x_2$.
$\hookrightarrow \Delta>0$

ALLORA:

$$\{x\in\mathbb{R} \mid x^2+bx+c>0\} = \{x\in\mathbb{R} \mid (x-x_1)\cdot(x-x_2)>0\} =$$

$$= \{x \in \mathbb{R} \mid x - x_1 > 0 \text{ e } x - x_2 > 0\} \cup$$

$$\{x \in \mathbb{R} \mid x - x_1 < 0 \text{ e } x - x_2 < 0\} =$$

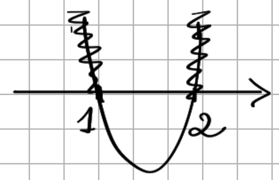
$$= \left(\{x \mid x - x_1 > 0\} \cap \{x \mid x - x_2 > 0\}\right) \cup \left(\{x \mid x - x_1 < 0\} \cap \{x \mid x - x_2 < 0\}\right)$$

$$= \left((x_1, +\infty) \cap (x_2, +\infty)\right) \cup \left((-\infty, x_1) \cap (-\infty, x_2)\right) =$$

$$\overset{x_1 < x_2}{=} (x_2, +\infty) \cup (-\infty, x_1) = (-\infty, x_1) \cup (x_2, +\infty)$$

ES. $x^2 - 3x + 2 > 0$

$(x-1)(x-2) > 0$



$(-\infty, 1) \cup (2, +\infty)$

ES. $x^2 - 4x - 5 < 0$

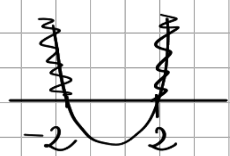
$(x-5)(x+1) < 0$

$x \in (-1, 5)$

$-1 < x < 5$

ES. $x^2 - 4 > 0$

$(x-2)(x+2) > 0$



$x < -2 \lor x > 2$

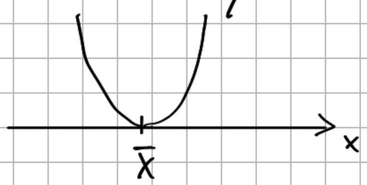
$(-\infty, -2) \cup (2, +\infty)$

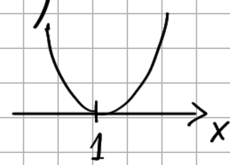
2) L'EQ. ASSOCIATA HA RADICI REALI COINCIDENTI $(\Delta = 0)$

$x_1 = x_2 = \bar{x}$

$x^2 + bx + c = (x - \bar{x})^2$

$(x - \bar{x})^2 > 0 \iff x \neq \bar{x} \quad (x \in \mathbb{R} \setminus \{\bar{x}\})$



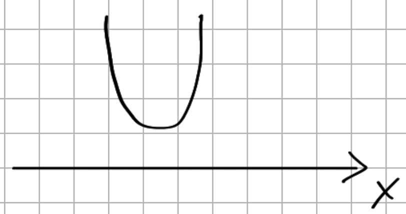


ES. $x^2 - 2x + 1 > 0 \iff (x-1)^2 > 0 \iff x \neq 1$

$x^2 - 2x + 1 \geq 0 \iff (x-1)^2 \geq 0 \quad \forall x \in \mathbb{R}$

3) L'EQ. ASSOCIATA NON HA RADICI REALI $(\Delta<0)$
(HA 2 RADICI COMPLESSE CONIUGATE).

$x_1, x_2 \notin \mathbb{R}$



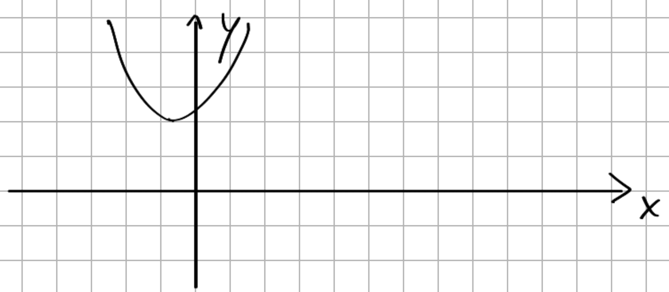
COMPLETANDO IL QUADRATO:

$$x^2+bx+c = \left(x^2+\frac{2bx}{2}\right)+c = \left(x^2+\frac{2bx}{2}+\frac{b^2}{4}\right)+c-\frac{b^2}{4} =$$

$$= \underbrace{\left(x+\frac{b}{2}\right)^2}_{\geq 0} + \underbrace{c-\frac{b^2}{4}}_{\frac{4c-b^2}{4}=\frac{-\Delta}{4}} \geq \frac{4c-b^2}{4} = -\frac{\Delta}{4} > 0$$

ES. $x^2+2x+5>0$

$\Delta = 4-4\cdot 5 = 4-20 = -16 < 0$



$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$

$V\left(-\frac{2}{2}, \frac{16}{4}\right)$

$V(-1, 4)$

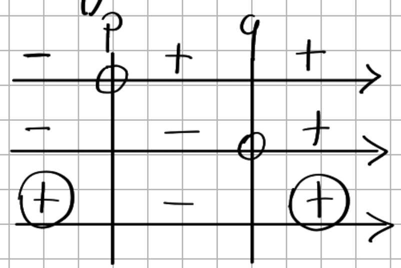
ABBIAMO PROVATO CHE, PER STUDIARE

$(x-p)(x-q)>0$ È SUFF. STUDIARE

$x-p>0$

$x-q>0$

$(x-p)\cdot(x-q)>0$



$p<q$

$x \in (-\infty, p) \cup (q, +\infty)$

OVVERO IL PRODOTTO DEI SEGNI.

ES. $2x^3+3x^2-2x-3>0 \iff$

$2x(x^2-1)+3(x^2-1)>0$

$(2x+3)(x^2-1)>0$

1. RACC. TOT.
2. RACC. PARZ.
3. PROD. NOT.
4. TRINOMIO PARTICOLARE
5. RUFFINI

$$(2x+3)(x-1)(x+1) > 0$$

$2x+3>0 \iff x > -\frac{3}{2}$

$x-1>0 \iff x>1$

$x+1>0 \iff x>-1$

	$-\infty$	$-3/2$ … -1	-1 … 1	1 … $+\infty$
F_1	$-$	$+$	$+$	$+$
F_2	$-$	$-$	$-$	$+$
F_3	$-$	$-$	$+$	$+$
$F_1 \cdot F_2 \cdot F_3$	$-$	$+$	$-$	$+$

$$-\frac{3}{2} < x < -1 \quad \vee \quad x > 1$$

$$x \in \left(-\frac{3}{2}, -1\right) \cup (1, +\infty)$$