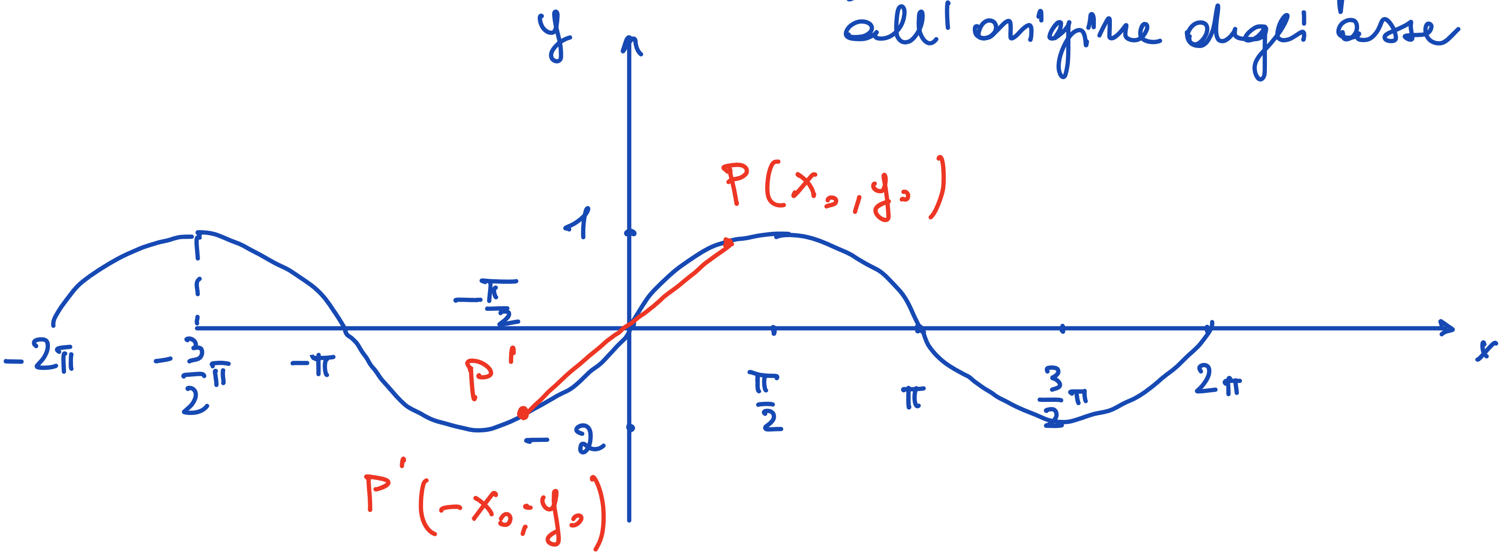
$y = \sin x$ periodica di periodo 2π

funzione dispari

$\sin(-x) = -\sin x$

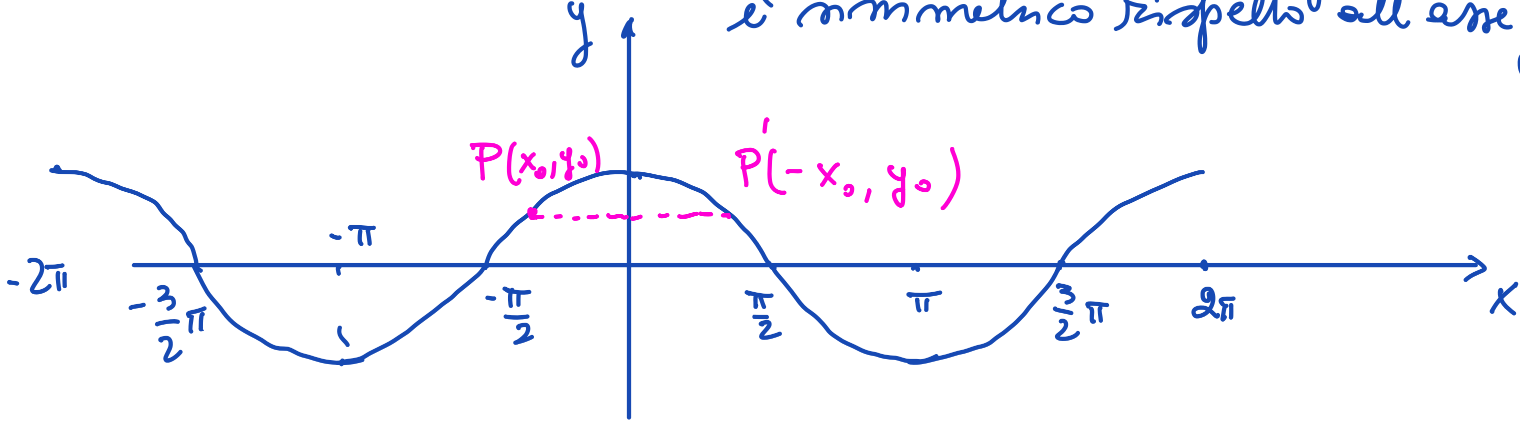
funzione il cui grafico è simmetrico rispetto all'origine degli assi



$\cos(-x) = \cos x$

funzione periodica di periodo 2π

funzione pari il cui grafico è simmetrico rispetto all'asse y



$\sin(x + 2k\pi) = \sin x$ $\quad k \in \mathbb{Z}$

$\cos(x + 2k\pi) = \cos x$

$y = \tan x$ periodica di periodo π

$\tan(x + k\pi) = \tan x$

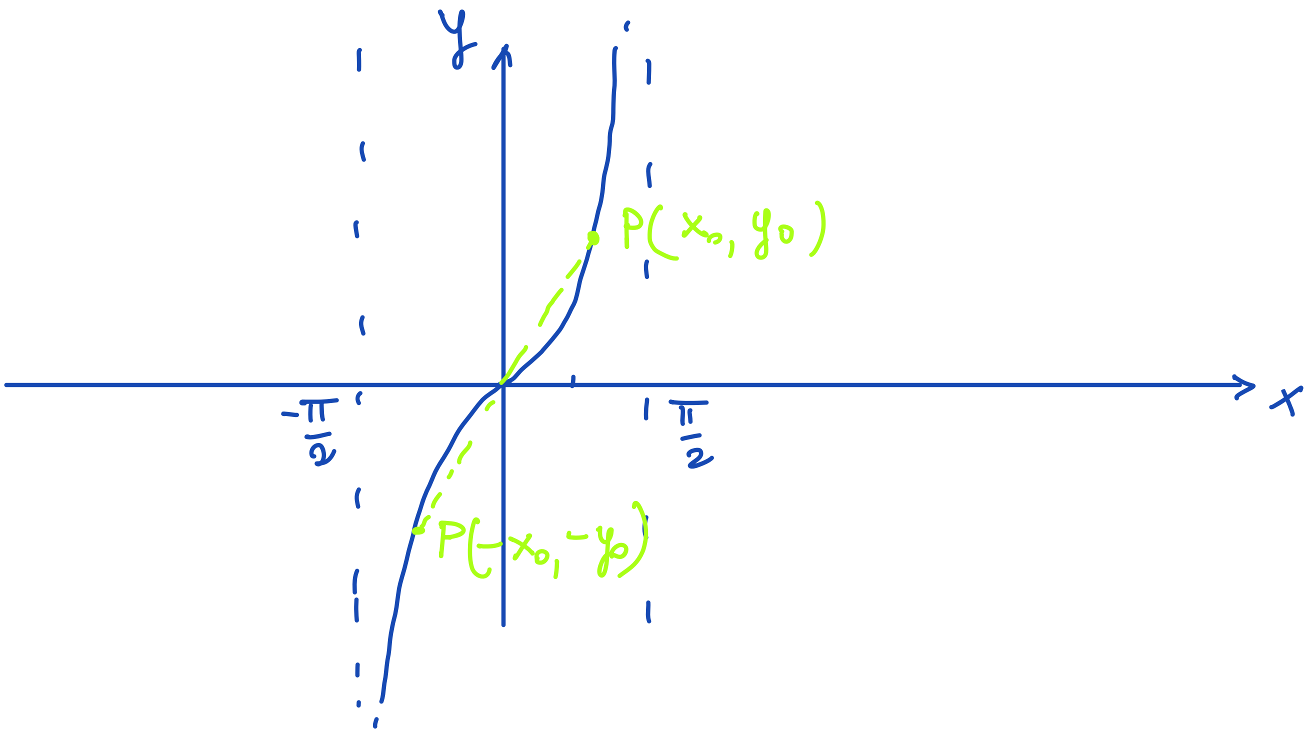
$$y = \frac{\sin x}{\cos x} = \tan x \qquad \cos x \neq 0$$

$x \neq \frac{\pi}{2}, \frac{3}{2}\pi, \ldots$

$x \neq \frac{\pi}{2} + k\pi$

$\tan(-x) = -\tan x$ funzione dispari.

Il suo grafico è simmetrico rispetto all'origine.

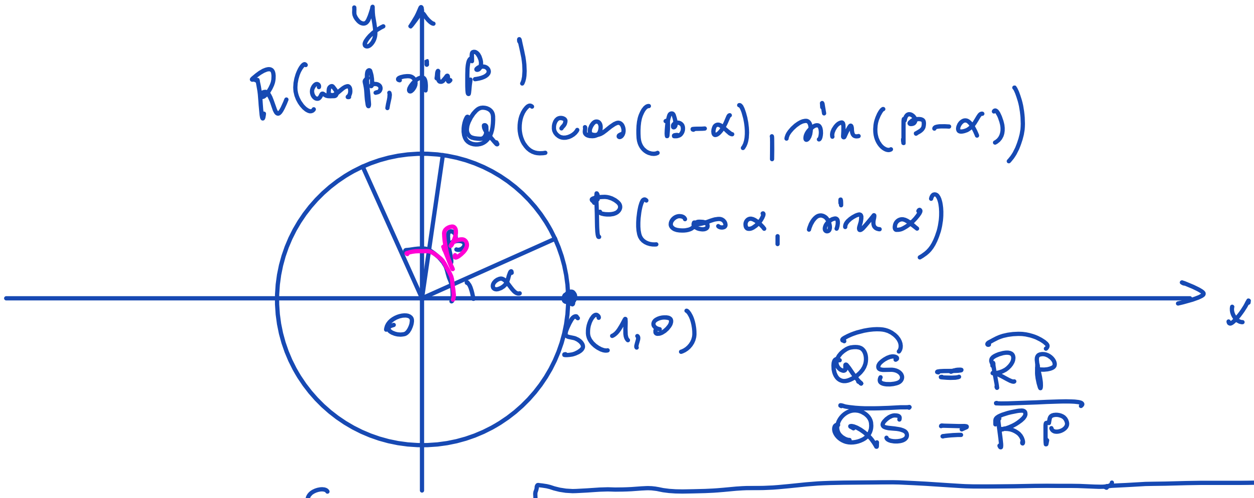


Formule di addizione

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \forall \alpha, \beta \in \mathbb{R}$

Dimostriamo che $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



$$\overset{\frown}{QS} = \overset{\frown}{RP}$$
$$\overline{QS} = \overline{RP}$$

$$d(Q, \overset{S}{(1,0)}) = \sqrt{(\cos(\beta-\alpha)-1)^2 + (\sin(\beta-\alpha)-0)^2}$$

$$\parallel$$

$$d(R, P) = \sqrt{(\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2} =$$

$$d(Q,S) = d(R,P) \Rightarrow d^2(Q,S) = d^2(R,P)$$

$$\overbrace{\cos^2(\beta-\alpha) + 1 - 2\cos(\beta-\alpha) + \sin^2(\beta-\alpha)}^{1} = \cos^2\beta + \cos^2\alpha +$$
$$-2\cos\beta\cos\alpha + \sin^2\beta + \sin^2\alpha - 2\sin\beta\sin\alpha$$

$$\cancel{1} + \cancel{1} - 2\cos(\beta-\alpha) = \cancel{1} + \cancel{1} - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha$$

Divido 1° e 2° m per (-2)

$$\boxed{\cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha}$$

$$\boxed{\begin{aligned}\cos(\beta+\alpha) &= \cos(\beta-(-\alpha)) = \cos\beta\cos(-\alpha) + \sin\beta\sin(-\alpha) \\ &= \cos\beta\cos\alpha - \sin\beta\sin\alpha\end{aligned}}$$

$$\boxed{\begin{aligned}\sin(\alpha+\beta) &= \cos\left(\frac{\pi}{2} - (\alpha+\beta)\right) = \cos\left(\left(\frac{\pi}{2}-\alpha\right) - \beta\right) = \\ &= \cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta + \sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta = \\ &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\end{aligned}}$$

$$\boxed{\begin{aligned}&\sin(\alpha-\beta) = \sin(\alpha+(-\beta)) = \sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta) =\\ &= \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{aligned}}$$

Formule di duplicazione

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

Dim:

$\sin(2x) = \sin(x+x) = \sin x\cos x + \sin x\cos x = 2\sin x\cos x$

$\cos(2x) = \cos(x+x) = \cos x\cos x - \sin x\sin x =$

$= \cos^2 x - \sin^2 x = \cos^2 x - (1-\cos^2 x) = \cos^2 x - 1 + \cos^2 x =$

$= 2\cos^2 x - 1 = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$

Formule di bisezione

$\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$ $\qquad$ $\cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$

$\cos x = \cos\left(2\cdot\frac{x}{2}\right) = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 1 - 2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} = 1 - \cos x$

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

$\cos x = 2\cos^2\frac{x}{2} - 1$ $\qquad$ $2\cos^2\frac{x}{2} = 1 + \cos x$

$\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$

$$\sin\frac{x}{2} = \begin{cases} \sqrt{\frac{1-\cos x}{2}} & 2k\pi \le \frac{x}{2} \le (2k+1)\pi \\ -\sqrt{\frac{1-\cos x}{2}} & (2k+1)\pi < \frac{x}{2} < (2k+2)\pi \end{cases}$$

$k \in \mathbb{Z}$

$$\cos\frac{x}{2} = \begin{cases} \sqrt{\frac{1+\cos x}{2}} & -\frac{\pi}{2} + 2k\pi \le \frac{x}{2} \le \frac{\pi}{2} + 2k\pi \\ -\sqrt{\frac{1+\cos x}{2}} & \frac{\pi}{2} + 2k\pi < \frac{x}{2} < \frac{3}{2}\pi + 2k\pi \end{cases}$$

$k \in \mathbb{Z}$

Esercizio: Calcolare $\sin\frac{\pi}{12}$ e $\cos\frac{\pi}{12}$

$$\sin\frac{\pi}{12} = \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} =$$

$$= \sqrt{\frac{4-2\sqrt{3}}{8}} = \sqrt{\frac{(\sqrt{3}-1)^2}{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos\frac{\pi}{12} = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \sqrt{\frac{4+2\sqrt{3}}{8}} =$$

$$= \sqrt{\frac{(\sqrt{3}+1)^2}{8}} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos x = \begin{cases} \sqrt{1-\sin^2 x} & \text{se } \cos x \ge 0 \\ -\sqrt{1-\sin^2 x} & \text{se } \cos x < 0 \end{cases}$$

$$\tan x = \frac{\sin x}{\cos x} = \begin{cases} \sqrt{\frac{\sin^2 x}{1-\sin^2 x}} & \text{se } \tan x > 0 \\ -\sqrt{\frac{\sin^2 x}{1-\sin^2 x}} & \text{se } \tan x < 0 \end{cases}$$

$$\sin x = \begin{cases} \sqrt{1-\cos^2 x} & \text{se } \sin x \geq 0 \\ -\sqrt{1-\cos^2 x} & \text{se } \sin x < 0 \end{cases}$$

$$\tan x = \frac{\sin x}{\cos x} = \begin{cases} \sqrt{\dfrac{1-\cos^2 x}{\cos^2 x}} & \text{se } \tan x > 0 \\ -\sqrt{\dfrac{1-\cos^2 x}{\cos^2 x}} & \text{se } \tan x < 0 \end{cases}$$

$$\sin x = \frac{\sin x}{\sqrt{\sin^2 x + \cos^2 x}} = \begin{cases} \sqrt{\dfrac{\sin^2 x}{\sin^2 x + \cos^2 x}} & \text{se } \tan x > 0 \\ -\sqrt{\dfrac{\sin^2 x}{\sin^2 x + \cos^2 x}} & \text{se } \tan x < 0 \end{cases}$$

$$= \begin{cases} \sqrt{\dfrac{\tan^2 x}{\tan^2 x + 1}} & \text{se } \tan x > 0 \\ -\sqrt{\dfrac{\tan^2 x}{\tan^2 x + 1}} & \text{se } \tan x < 0 \end{cases}$$

$$\cos x = \frac{\cos x}{\sqrt{\sin^2 x + \cos^2 x}} = \begin{cases} \sqrt{\dfrac{\cos^2 x}{\sin^2 x + \cos^2 x}} & \text{se } \cos x \geq 0 \\ -\sqrt{\dfrac{\cos^2 x}{\sin^2 x + \cos^2 x}} & \text{se } \cos x < 0 \end{cases}$$

$$= \begin{cases} \sqrt{\dfrac{1}{\tan^2 x + 1}} & \text{se } \cos x \geq 0 \\ -\sqrt{\dfrac{1}{\tan^2 x + 1}} & \text{se } \cos x < 0 \end{cases}$$

Formule parametriche

esprimono $\sin x$, $\cos x$ e $\tan x$ in funzione di $\tan\frac{x}{2}$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \tan x = \frac{2t}{1-t^2}$$

$$t = \tan\frac{x}{2}$$

$$\sin x = \sin 2\left(\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}} =$$

$\cos\frac{x}{2} \neq 0 \Rightarrow \frac{x}{2} \neq \frac{\pi}{2} + k\pi \Rightarrow x \neq \pi + 2k\pi$

$$= \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \boxed{\frac{2t}{1+t^2}}$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}} =$$

$\cos\frac{x}{2} \neq 0 \Rightarrow \frac{x}{2} \neq \frac{\pi}{2} + k\pi \Rightarrow x \neq \pi + 2k\pi$

$$= \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \boxed{\frac{1-t^2}{1+t^2}}$$

tangente di una somma

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} \qquad \forall \alpha, \beta : \tan\alpha\tan\beta \neq 1$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} =$$

$$= \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\beta\cos\alpha}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$$

Formule di Prostaferesi

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\sin\alpha = \sin\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right) = \sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + \sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\sin\beta = \sin\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right) = \sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} - \sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\boxed{\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}$$