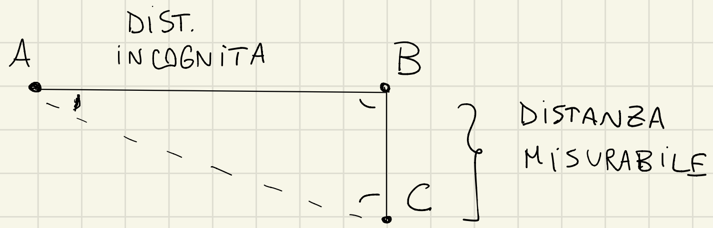
12/9/22

Trigonometria parte 1

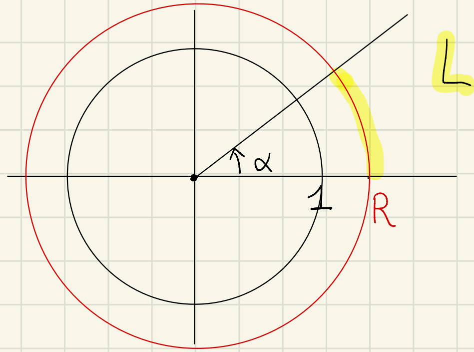
PROBLEMA: COME CALCOLARE LA DISTANZA DI UN OGGETTO INACCESSIBILE A A PARTIRE DA UNA POSIZIONE ACCESSIBILE B?



- INTRODUCO UN SECONDO PT ACCESSIBILE C.
- MISURO $\overline{BC}$, $A\hat{B}C$, $B\hat{C}A$.
- $C\hat{A}B = \pi - A\hat{B}C - B\hat{C}A$
- SE $AB \perp BC$, $d(A,B) = d(B,C) \cdot \tan(C\hat{A}B)$.

GLI ANGOLI SONO ESPRESSI IN RADIANTI:

DATO UN ANGOLO α E FISSATO UN RAGGIO R, SI TRACCI LA PORZIONE DI CIRCONFERENZA DI RAGGIO R SOTTESA DA α. DENOTIAMO QUEST'ULTIMA CON L. $\frac{L}{R}$ E' IL VALORE IN RADIANTI DI α.



R = 1 ⇒ IL VALORE IN RADIANTI DI α

E' PARI A L.

NE SEGUE LA SEGUENTE CORRISPONDENZA:

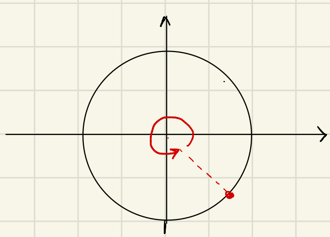
GRADI	RADIANTI	GRADI	RADIANTI
$30°$	$\pi/6$	$210°$	$7/6\,\pi$
$45°$	$\pi/4$	$225°$	$5/4\,\pi$
$60°$	$\pi/3$	$240°$	$4/3\,\pi$
$90°$	$\pi/2$	$270°$	$3/2\,\pi$
$120°$	$2\pi/3$	$300°$	$5/3\,\pi$
$135°$	$3\pi/4$	$315°$	$7/4\,\pi$
$150°$	$5\pi/6$	$340°$	$11/6\,\pi$
$180°$	π	$360°$	2π

ANGOLO IN GRADI : ANGOLO IN RAD = $360°$: 2π

Es. TRADURRE IN RADIANTI: $-45°$, $105°$

$$-45° = 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi$$

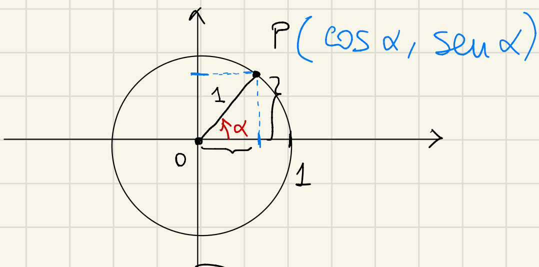
$$105° = \frac{7}{12}\pi = \frac{\pi}{2} + \frac{1}{2}\cdot\frac{\pi}{6}$$



Es. Tradurre in gradi: $-\frac{\pi}{6}, \frac{7\pi}{2}, \frac{3\pi}{4}, \frac{\pi}{12}$.

$330° \quad 630° \quad 135° \quad 15°$.

Def di sen(x) e cos(x)



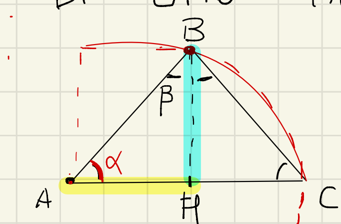
Dal teorema di Pitagora:

$$\cos^2\alpha + \sin^2\alpha = 1$$

Relazione fondamentale.

Calcolo di sen x e cos x per angoli noti

- Considero il triangolo equilatero $\hat{ABC}$ di lato pari a 1.



$$\overline{BH}^2 + \overline{HC}^2 = \overline{BC}^2$$
$$\overline{BH}^2 + \frac{1}{4} = 1$$

$\hat{A} = \hat{B} = \hat{C} = \frac{\pi}{3}$, $\quad A\hat{B}H = H\hat{B}C = \frac{\pi}{6}$,

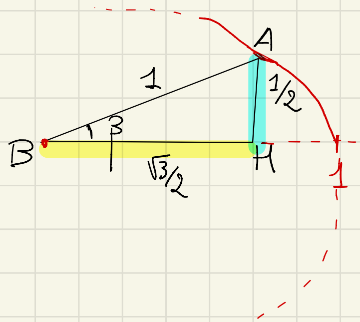
$\overline{AH} = \overline{HC} = \frac{1}{2}$, $\quad \overline{BH} = \frac{\sqrt{3}}{2}$

Pitagora

NE SEGUE CHE, PER $\alpha = \frac{\pi}{3}$,

$$\cos\alpha = \overline{AH} = \frac{1}{2}.$$

$$\sin\alpha = \overline{BH} = \frac{\sqrt{3}}{2}$$



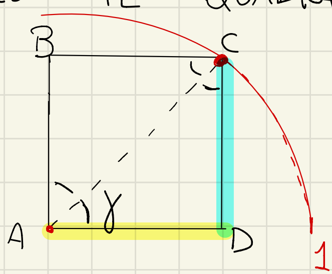
PER $\beta = \frac{\pi}{6}$:

$$\cos\beta = \overline{BH} = \frac{\sqrt{3}}{2}$$

$$\sin\beta = \overline{AH} = \frac{1}{2}$$

- CONSIDERIAMO IL QUADRATO DI LATO $\frac{1}{\sqrt{2}}$.

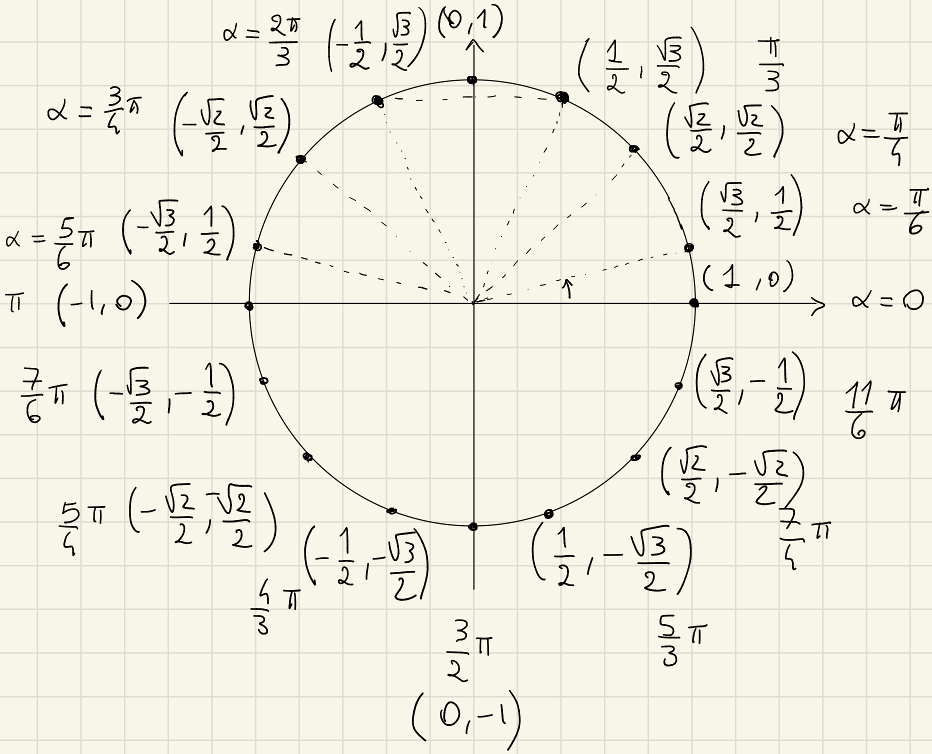
$\overline{AC} = 1$.



$\gamma = \frac{\pi}{4}$.

$$\cos\gamma = \overline{AD} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

$$\sin\gamma = \overline{CD} = \frac{\sqrt{2}}{2}.$$



Oss.

$0 \le \alpha \le \frac{\pi}{2} \Rightarrow 0 \le \operatorname{sen}\alpha \le 1 \quad 0 \le \cos\alpha \le 1$

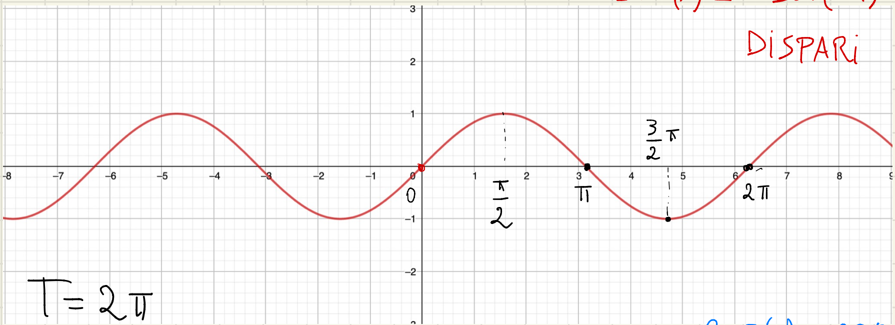
$\frac{\pi}{2} < \alpha \le \pi \Rightarrow 0 < \operatorname{sen}\alpha < 1 \quad -1 < \cos\alpha < 0$

$\pi < \alpha < \frac{3}{2}\pi \Rightarrow -1 < \operatorname{sen}\alpha < 0 \quad -1 < \cos\alpha < 0$

$\frac{3}{2}\pi < \alpha < 2\pi \Rightarrow -1 < \operatorname{sen}\alpha < 0 \quad 0 < \cos\alpha < 1.$

$y = \sin x$

$\sin(x) = -\sin(-x)$



$y = \cos x$

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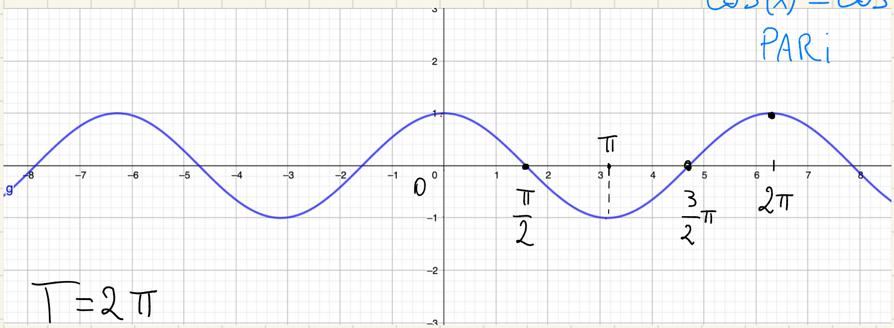
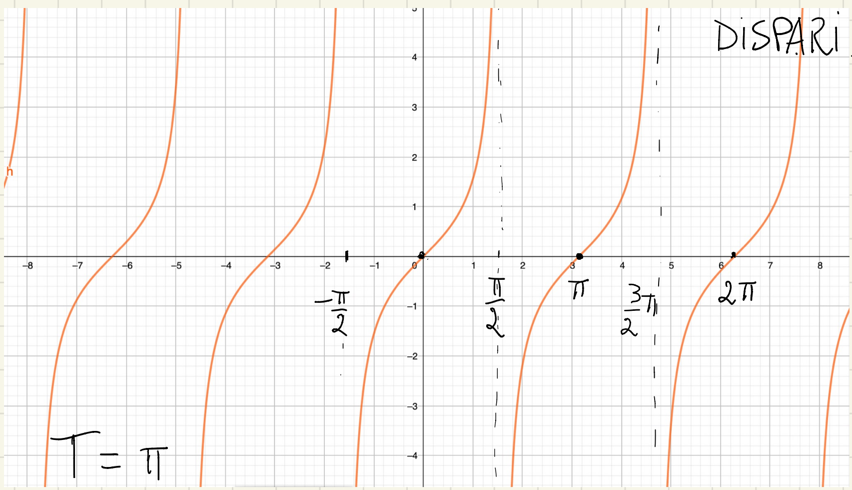


$y = \tan x$



Es.

$$\operatorname{sen}\left(x+\frac{\pi}{2}\right) = \cos x$$

$$\operatorname{sen}\left(x-\frac{\pi}{2}\right) = -\cos x$$

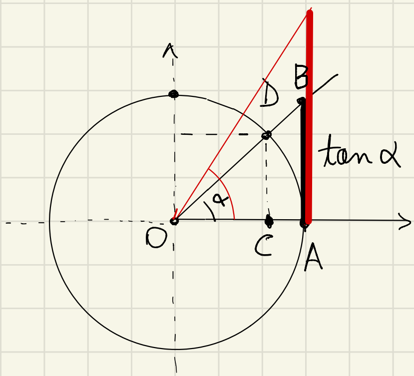
$$\cos\left(x+\frac{\pi}{2}\right) = -\sin x$$

$$\cos\left(x-\frac{\pi}{2}\right) = \sin x$$

1 $\cos x$ — 2 $-\cos x$

3 $\sin x$ — 3 $-\sin x$

DEF.



$\overline{OC} = \cos\alpha$

$\overline{CD} = \sin\alpha$

$\overset{\triangle}{OAB}$ E $O\hat{C}D$ SONO SIMILI:

$$\Rightarrow \frac{AB}{OA} = \frac{CD}{OC} \Rightarrow \frac{\tan\alpha}{1} = \frac{\sin\alpha}{\cos\alpha}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\alpha \neq \frac{\pi}{2} \quad !!!$$

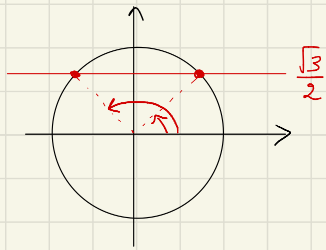
ES. DETERMINARE i VALORI DI x PER CUI
SI HA:

ⓐ $\operatorname{sen} x = \frac{\sqrt{3}}{2}$ ⓑ $\cos x \leq \frac{1}{2}$.

ⓐ $x = \frac{\pi}{3} + 2k\pi \, , \, k \in \mathbb{Z}$

$\vee$

$x = \frac{2\pi}{3} + 2k\pi \, , \, k \in \mathbb{Z}$



ⓑ

$\frac{\pi}{3} + 2k\pi \leq x \leq \frac{5}{3}\pi + 2k\pi \, ,$

$k \in \mathbb{Z}$.