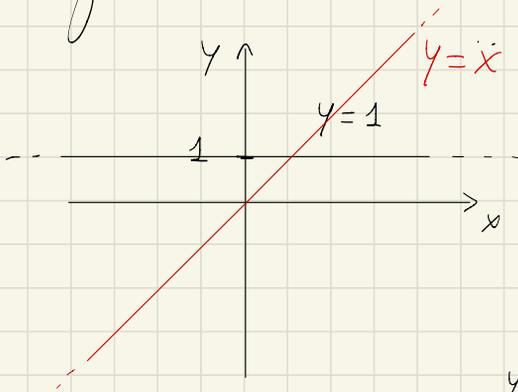


Quarta lezione - seconda parte - 8/9/23

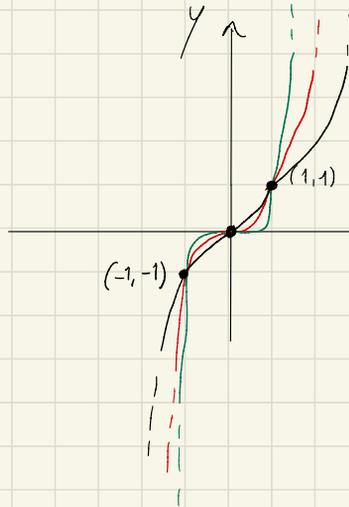
Grafici di funzioni e trasformazioni di grafici

POLINOMI $1, x, x^2, x^3, \dots$

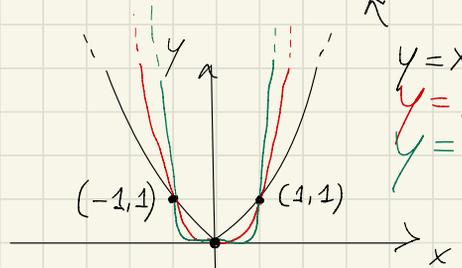
$f: \mathbb{R} \rightarrow \mathbb{R}$



$y = x^5$
 $y = x^7$



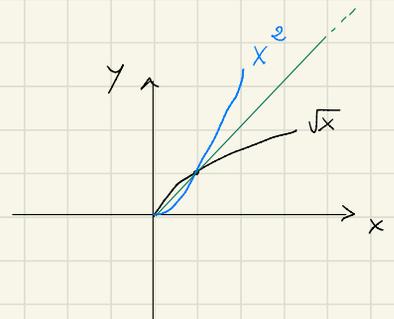
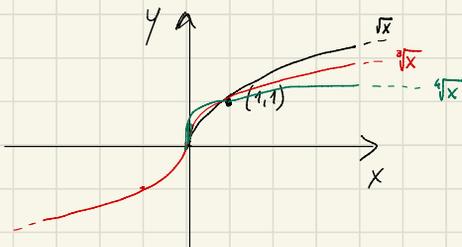
PARI \rightarrow SIM. RISP. ASSE Y
 $y = x^2$
 $y = x^4$
 $y = x^6$



$f(x) = f(-x)$

$y = x^3$
SIM. RISP. O
FE. DISPARI
 $f(x) = -f(-x)$

RADICI



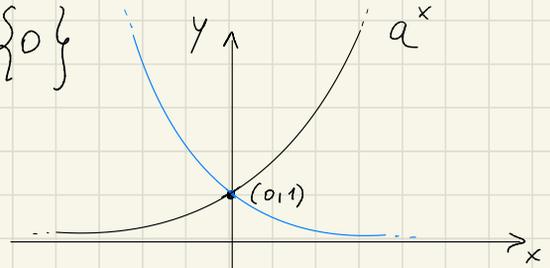
Dominio: $\sqrt{x} \quad x \geq 0$
 $\sqrt[3]{x} \quad \mathbb{R}$

M PARI: $f(x) = \sqrt[n]{x}$
 $f(x): \mathbb{R}^+ \rightarrow \mathbb{R}^+$

M DISPARI: $f(x) = \sqrt[n]{x}$
 $f(x): \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = a^x: \mathbb{R} \rightarrow \mathbb{R}^+ \setminus \{0\}$
 $a \in \mathbb{R}^+ \setminus \{0\}$

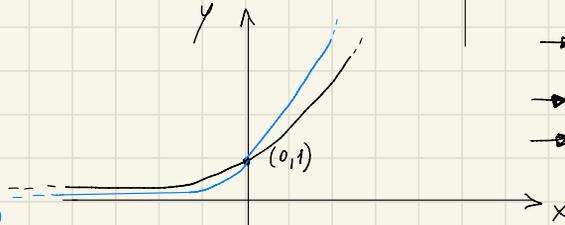
$a > 1$
 $0 < a < 1$



$f(x) = e^x$

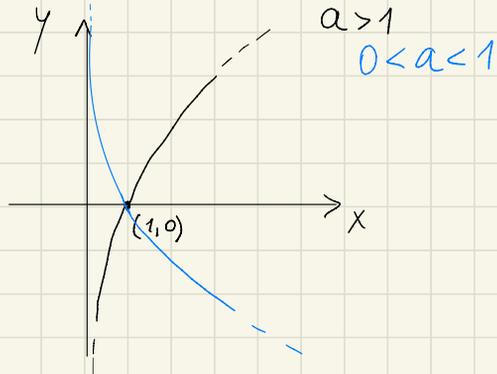
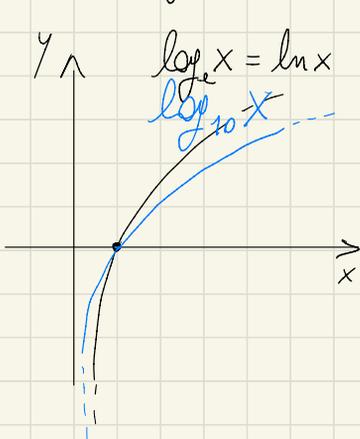
$f(x) = 10^x$

$10 > e \approx 2,72$

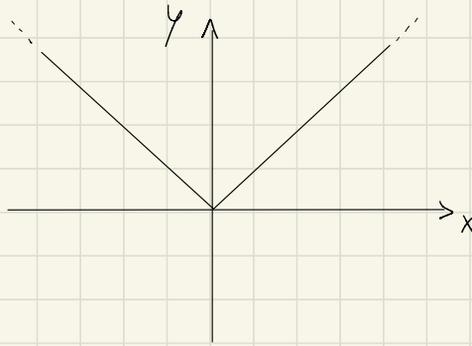


- $f(0) = 1$.
- POSITIVA
- CRESCENTE

$f(x) = \log_a x: \mathbb{R}^+ \setminus \{0\} \rightarrow \mathbb{R}$

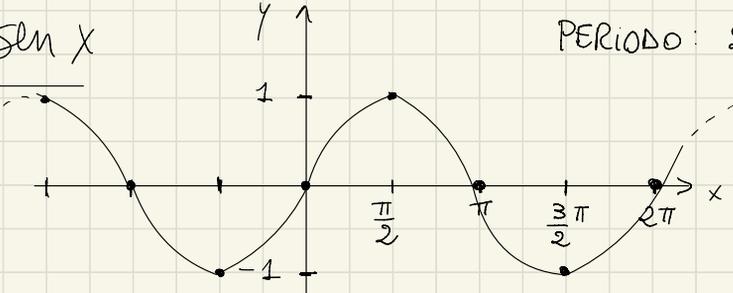


$$f(x) = |x|$$



$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$f(x) = \sin x$$



PERIODO: 2π

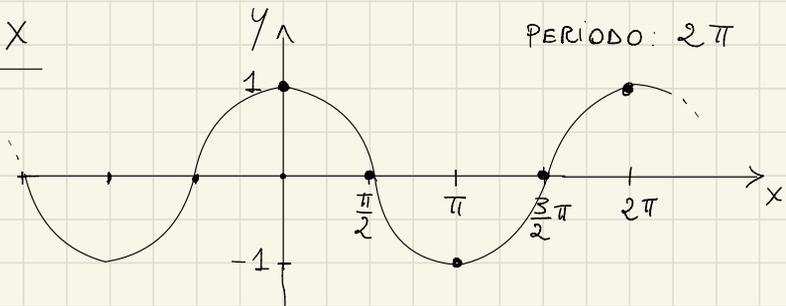
→ PARI/DISPARI ✓

$$\sin x = -\sin(-x)$$

→ D: \mathbb{R}

$$Im: [-1, 1]$$

$$f(x) = \cos x$$



PERIODO: 2π

→ PARI $\cos x = \cos(-x)$

→ D: \mathbb{R}

$$Im: [-1, 1]$$

ES TROVARE IL DOMINIO DELLE SEGUENTI FUNZIONI:

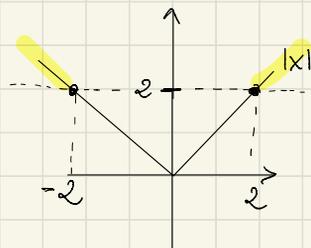
- 1) $y = \sqrt{x-2}$ 2) $y = \sqrt{|x-2|}$
3) $y = \sqrt{|x|-2}$ 4) $y = \sqrt{\ln x + 1}$
5) $y = \ln(\sqrt{x^2 - 6x + 5})$
6) $y = \sin(x - \sqrt{1-2x})$

1) $x - 2 \geq 0 \Rightarrow x \geq 2.$

2) $|x - 2| \geq 0 \quad \forall x \in \mathbb{R}$

3) $|x| - 2 \geq 0 \Leftrightarrow |x| \geq 2$

$\Leftrightarrow x \leq -2 \vee x \geq 2$



4) $\ln x + 1 \geq 0 \Leftrightarrow \ln x \geq -1 = -\ln e =$
 $= 0 - \ln e =$
 $= \ln 1 - \ln e =$
 $= \ln\left(\frac{1}{e}\right)$
 $\stackrel{e > 1}{\Leftrightarrow} x \geq \frac{1}{e}$

5) $\left. \begin{array}{l} \bullet \sqrt{x^2 - 6x + 5} > 0 \\ \bullet x^2 - 6x + 5 \geq 0 \end{array} \right\} x^2 - 6x + 5 > 0$

$\Leftrightarrow \underbrace{(x-5)}_{F_1} \underbrace{(x-1)}_{F_2} > 0 \Leftrightarrow x < 1 \vee x > 5.$

6) $1 - 2x \geq 0 \Leftrightarrow x \leq \frac{1}{2}.$

TRASFORMAZIONI DI GRAFICI

$$y = f(x)$$

• TRASLAZIONE

ORIZZONTALE
VERTICALE

↳ VERSO DX
↳ VERSO SX

$$y = f(x-a), a > 0$$

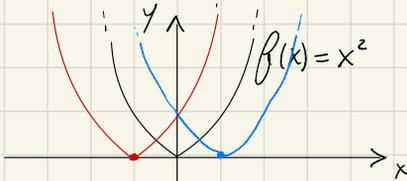
$$y = f(x+a), a > 0$$

↳ VERSO ALTO

↳ VERSO BASSO

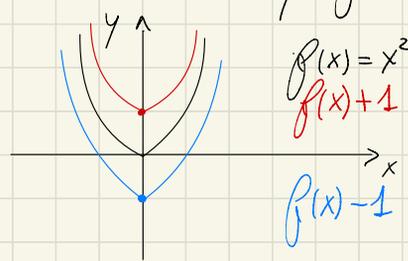
$$y = f(x)+b, b > 0$$

$$y = f(x)-b, b > 0$$



$$f(x-1) = (x-1)^2$$

$$f(x+1) = (x+1)^2$$



$$f(x) = x^2$$

$$f(x)+1$$

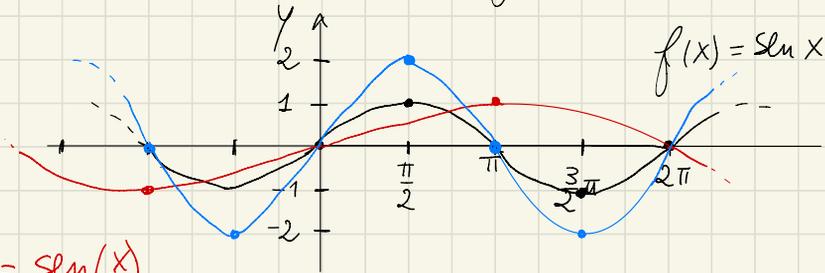
$$f(x)-1$$

• DILATAZIONI

ORIZZONTALI
VERTICALI

$$f(Kx) \quad 0 < K < 1$$

$$Kf(x) \quad K > 1$$



$$f\left(\frac{1}{2}x\right) = \sin\left(\frac{x}{2}\right)$$

$$2f(x) = 2\sin(x)$$

$$x = \frac{\pi}{2} \Rightarrow f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$$

$$\Rightarrow 2f\left(\frac{\pi}{2}\right) = 2\sin\frac{\pi}{2} = 2$$

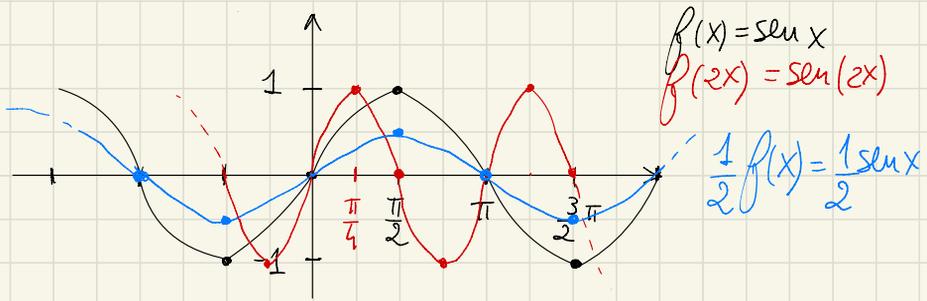
$$x = \pi \Rightarrow 2 \cdot \sin\pi = 0$$

• CONTRAZIONI

ORIZZONTALI
VERTICALI

$$f(Kx) \quad K > 1$$

$$Kf(x) \quad 0 < K < 1$$



$$x = \frac{\pi}{2} \Rightarrow f(x) = 1$$

$$\Rightarrow f(2x) = \sin\left(2 \cdot \frac{\pi}{2}\right) = \sin(\pi) = 0$$

$$x = \frac{\pi}{2} \Rightarrow f(x) = 1$$

$$\Rightarrow \frac{1}{2} f(x) = \frac{1}{2}$$

ES. GRAFICO Di $f(x) = \frac{1}{4} \cos(2x)$