

EQUAZIONI E DISEQ. EXP/LOG

$$1) 10^x = 100 \quad x = 2$$

$$2) 7^x = 1 \quad x = 0$$

$$3) 9^{x+2} = \sqrt[3]{3^{x+7}}$$

$$(3^2)^{x+2} = (3^{x+7})^{\frac{1}{3}} \Leftrightarrow 3^{2x+4} = 3^{\frac{x+7}{3}}$$

$$\Leftrightarrow 2x+4 = \frac{x+7}{3} \Leftrightarrow 2x - \frac{x}{3} = \frac{7}{3} - 4$$

$$\Leftrightarrow \frac{5}{3}x = \frac{-5}{3} \Leftrightarrow x = -1$$

$$4) 3 \cdot 5^x + 5^{x+1} = 8 \cdot 5^3$$

$$3 \cdot 5^x + 5^x \cdot 5 = 8 \cdot 5^3$$

$$5^x (3+5) = 8 \cdot 5^3 \Leftrightarrow x = 3$$

$$5) 9^x - 3 = 2 \cdot 3^x$$

$$3^{2x} - 3 = 2 \cdot 3^x \Leftrightarrow 3^{2x} - 2 \cdot 3^x - 3 = 0$$

$$t := 3^x$$

$$t^2 - 2t - 3 = 0$$

$$t_{1,2} = \left\langle \begin{array}{l} -1 \\ 3 \end{array} \right.$$

$$3^x \neq -1$$

$$\vee 3^x = 3 \Leftrightarrow x = 1$$

IMPOS.

$$6) 10^x + 10^{2-x} = 101$$

$$10^x + \frac{10^2}{10^x} = 101$$

$$10^x =: t$$

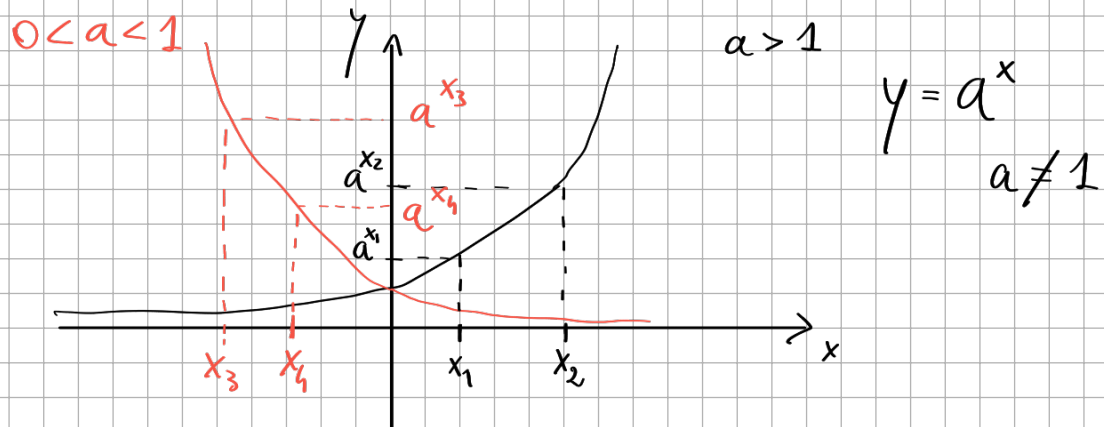
$$t + \frac{10^2}{t} = 101 \stackrel{t \neq 0}{\Leftrightarrow} t^2 + 10^2 - 101t = 0$$

$$\Leftrightarrow t^2 - 101t + 100 = 0$$

$$\Delta = 99^2 \quad \sqrt{\Delta} = 99$$

$$t_{1,2} = \frac{101 \pm 99}{2} = \left\langle \begin{array}{l} 1 \\ 100 \end{array} \right.$$

$$10^x = 1 \quad \vee \quad 10^x = 100 \quad \Leftrightarrow \quad x=0 \quad \vee \quad x=2.$$



$$a^{x_1} < a^{x_2} \quad \Leftrightarrow \quad x_1 < x_2$$

$$a^{x_3} > a^{x_4} \quad \Leftrightarrow \quad x_3 < x_4$$

↑

ES.

$$1) \quad 4^x \leq 32 \quad \Leftrightarrow \quad 2^{2x} \leq 2^5 \quad \begin{matrix} 2 > 1 \\ \Leftrightarrow \end{matrix} \quad 2x \leq 5$$

$$\Leftrightarrow x \leq \frac{5}{2}$$

$$2) \quad \left(\frac{1}{4}\right)^{x-1} < 64 \quad \Leftrightarrow \quad 4^{-(x-1)} < 4^3$$

$$\Leftrightarrow -x+1 < 3 \quad \Leftrightarrow -x < 2 \quad \Leftrightarrow x > -2$$

$$3) \quad \left(\frac{1}{5}\right)^{2x+1} < 625 \quad \Leftrightarrow \quad 5^{-2x-1} < 5^4$$

$$\Leftrightarrow -2x-1 < 4 \quad \Leftrightarrow -2x < 5$$

$$\Leftrightarrow x > -\frac{5}{2}$$

$$4) \quad 2^x \cdot 3^{x+1} \leq \frac{6^{3x}}{2} \quad \Leftrightarrow \quad 2^x \cdot 3^x \cdot 3 \leq \frac{6^{3x}}{2}$$

$$\Leftrightarrow \cancel{6^x} \cdot 3 \leq \frac{6^{3x}}{2} \cdot \frac{1}{\cancel{6^x}} \quad \begin{matrix} 6^x \neq 0 \\ \Leftrightarrow \end{matrix} \quad 2 \cdot 3 \leq \frac{6^{3x}}{6^x} \cdot \frac{1}{2}$$

$$\Leftrightarrow 6^1 \leq 6^{2x} \quad \Leftrightarrow 1 \leq 2x$$

$$\Leftrightarrow x \geq \frac{1}{2}$$

$$1) \log_3 x = 3, x > 0 \quad [\log_a b = y \Leftrightarrow a^y = b]$$

$$\Leftrightarrow x = 3^3 = 27$$

$$2) \log_3 x = \log_3 2 - \log_3 (x+1)$$

$$\begin{cases} x > 0 \\ x+1 > 0 \end{cases} \text{ C.E.} \Leftrightarrow \begin{cases} x > 0 \\ x > -1 \end{cases} \Leftrightarrow x > 0$$

$$\log_3 x = \log_3 \left(\frac{2}{x+1} \right)$$

$$(x+1) \cdot x = \frac{2}{x+1} \quad (x+1) \Leftrightarrow x(x+1) = 2 \Leftrightarrow x^2 + x - 2 = 0$$

$$\Leftrightarrow (x+2)(x-1) = 0 \quad \stackrel{\text{LAP}}{\Leftrightarrow} x+2=0 \vee x-1=0$$

$$\Leftrightarrow x = -2 \vee x = 1$$

NON ACCETTABILE

$$3) \log_2 x + \log_4 x = 3 \quad x > 0$$

$$\log_4 x = \frac{\log_2 x}{\log_2 4} = \frac{\log_2 x}{2}$$

$$\log_2 x + \frac{\log_2 x}{2} = 3 \Leftrightarrow \frac{2}{2} \frac{3}{2} \log_2 x = 3 \cdot \frac{2}{2}$$

$$\Leftrightarrow \log_2 x = 2 \Leftrightarrow x = 2^2 = 4.$$

$$4) 4 \log_4 x - \log_2 (1+x) = 0 \quad x > 0$$

$$\frac{4}{2} \log_2 x - \log_2 (1+x) = 0$$

$$\log_2 x^2 - \log_2 (1+x) = 0$$

$$\log_2 \left(\frac{x^2}{1+x} \right) = 0 = \log_2 1$$

$$(1+x) \cdot \frac{x^2}{1+x} = 1(1+x) \Leftrightarrow x^2 - (1+x) = 0$$

$$\Leftrightarrow x^2 - x - 1 = 0 \quad \Leftrightarrow$$

$$\Delta = 1 - 4 \cdot (-1) = 5$$

$$x_{1,2} = \frac{1 \pm \sqrt{5}}{2} = \begin{cases} \frac{1+\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} < 0 \text{ N.A.} \end{cases}$$

DISEQ. LOG.

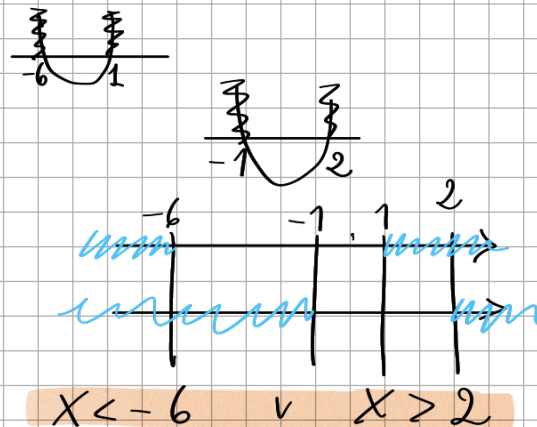
$$\log_a x > \log_a y \quad \begin{matrix} a > 1 \\ \rightarrow \\ x > y \\ 0 < a < 1 \\ \rightarrow \\ x < y \end{matrix}$$

ES. $\ln(x^2 + 5x - 6) > \ln(3x^2 - 3x - 6)$

$$\text{C.E.} \begin{cases} x^2 + 5x - 6 > 0 \\ 3x^2 - 3x - 6 > 0 \end{cases} \Leftrightarrow \begin{cases} (x+6)(x-1) > 0 \\ 3(x^2 - x - 2) > 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} (x+6)(x-1) > 0 \\ 3(x-2)(x+1) > 0 \end{cases}$$

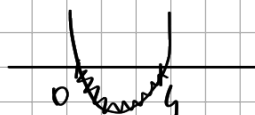
$$\Leftrightarrow \begin{cases} x < -6 \vee x > 1 \\ x < -1 \vee x > 2 \end{cases}$$



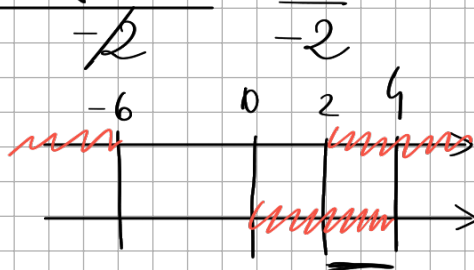
$$x^2 + 5x - 6 > 3x^2 - 3x - 6$$

$$-2x^2 + 8x > 0 \quad \Leftrightarrow \quad -2x(x-4) > 0$$

$$x(x-4) < 0$$



$$0 < x < 4$$



$$2 < x < 4$$