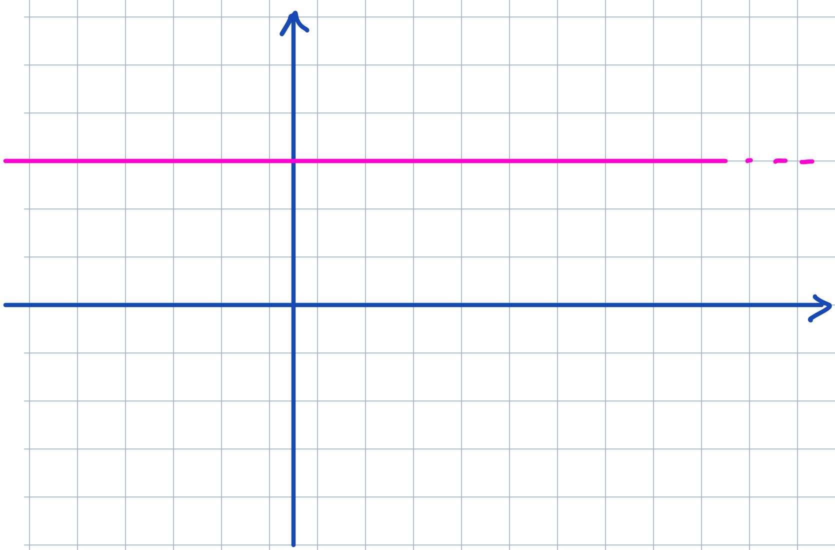
GRAFICI FUNZIONI ELEMENTARI

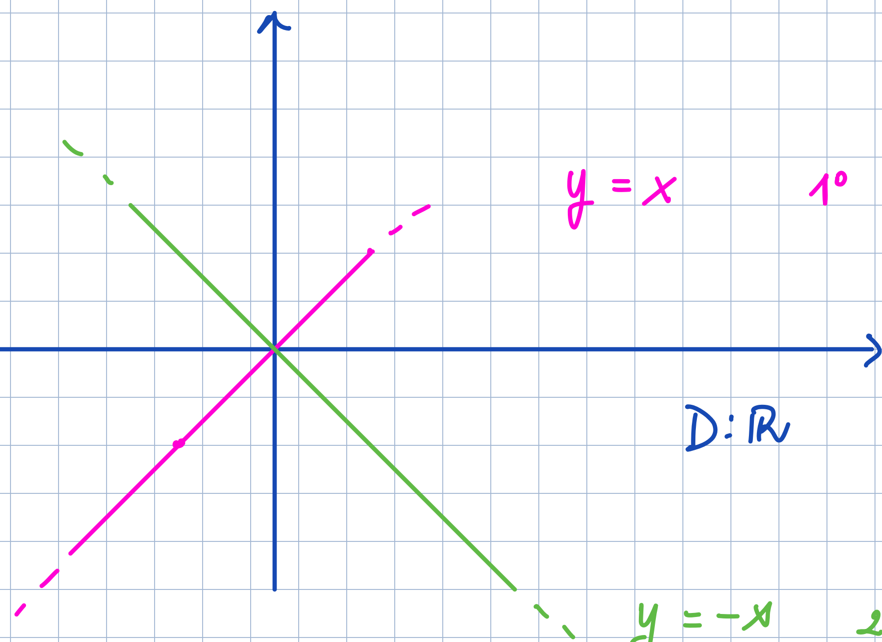
FUNZIONE COSTANTE



$y = K$

$D: \mathbb{R}$

BISETTRICE

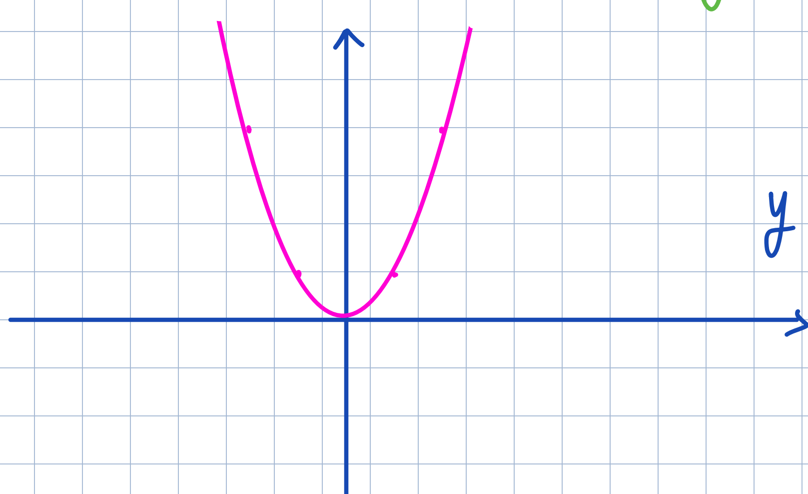


$y = x$ 1° E 3°

$D: \mathbb{R}$

$y = -x$ 2° E 4°

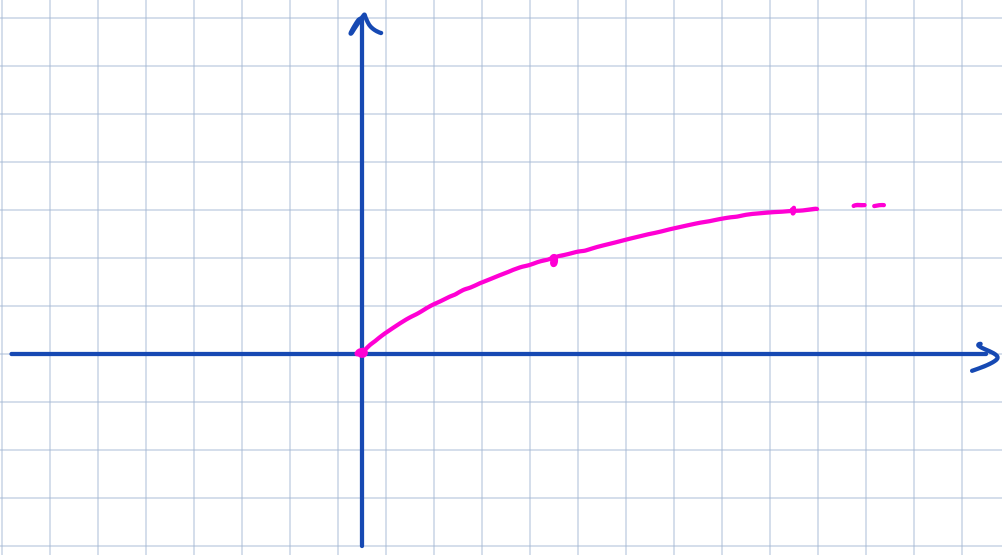
PARABOLA



$y = x^2$

$D: \mathbb{R}$

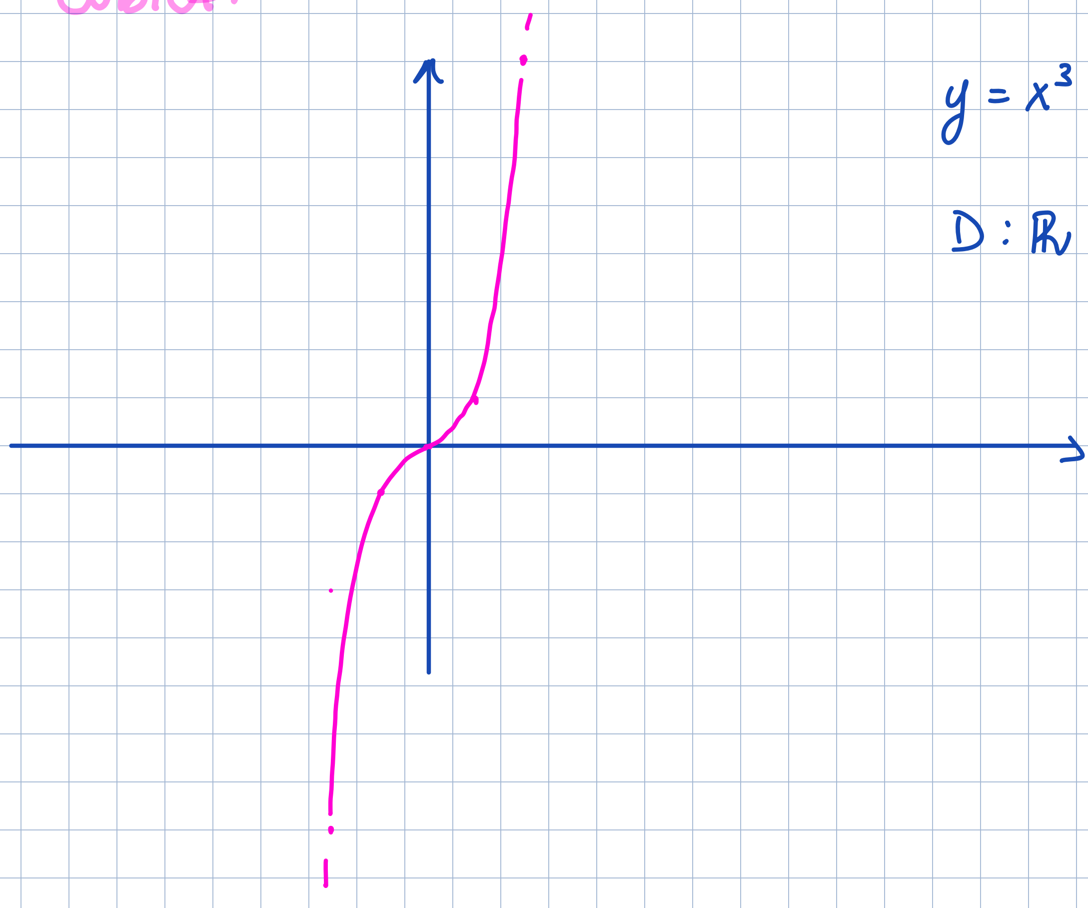
FUNZIONE RADICE CON INDICE PARI



$y = \sqrt{x}$

$D: [0; +\infty)$

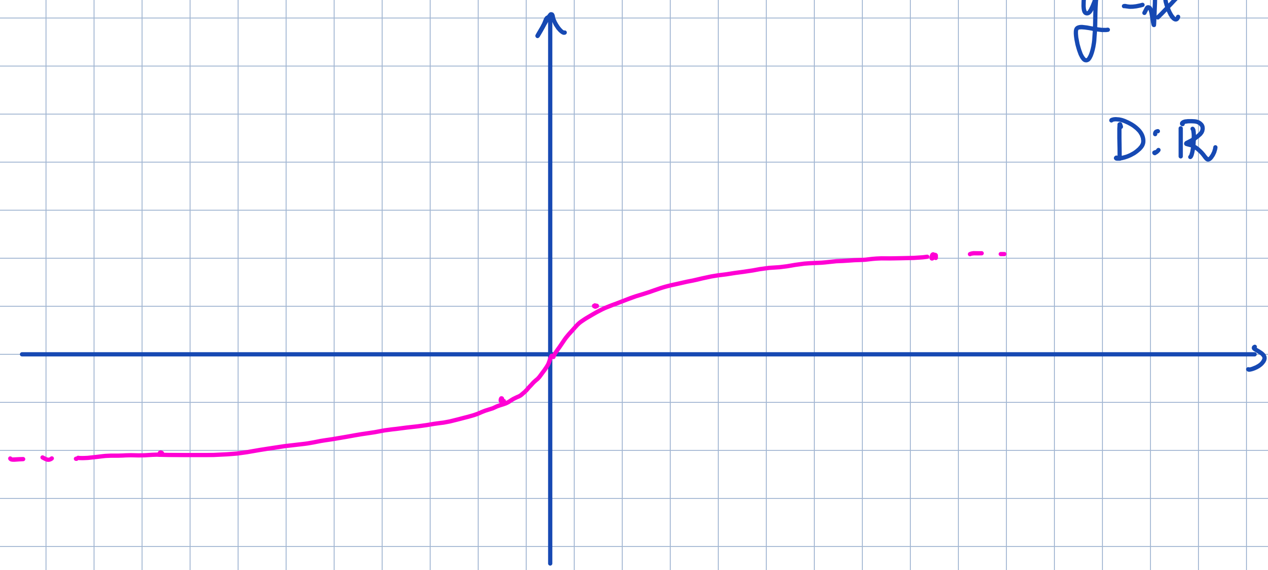
FUNZIONE CUBICA:



$y = x^3$

$D: \mathbb{R}$

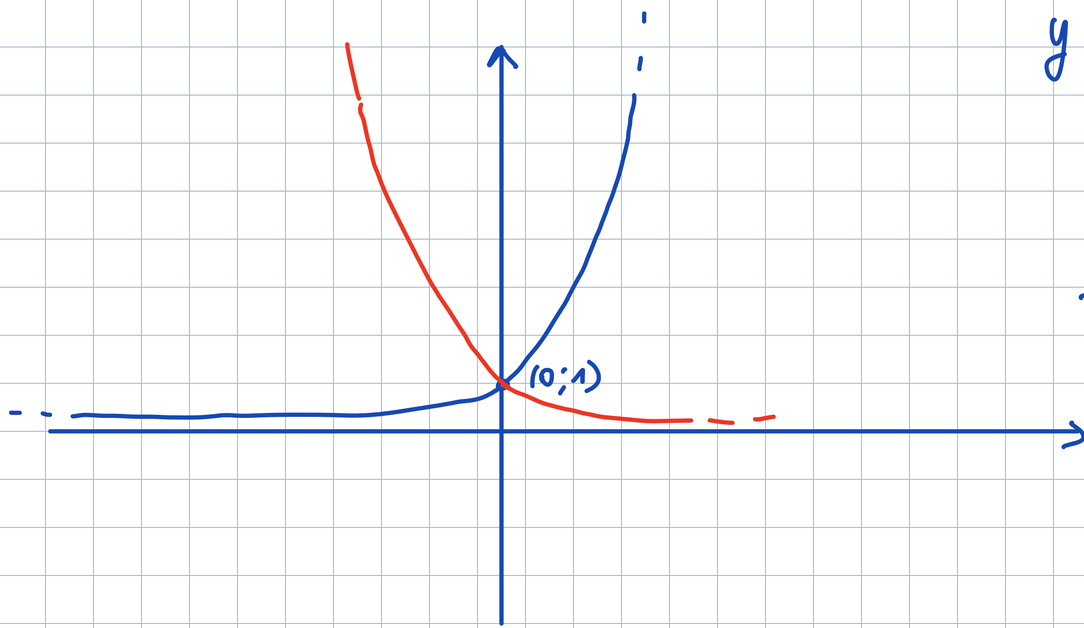
FUNZIONE RADICE CUBICA:



$y = \sqrt[3]{x}$

$D: \mathbb{R}$

FUNZIONE ESPONENZIALE



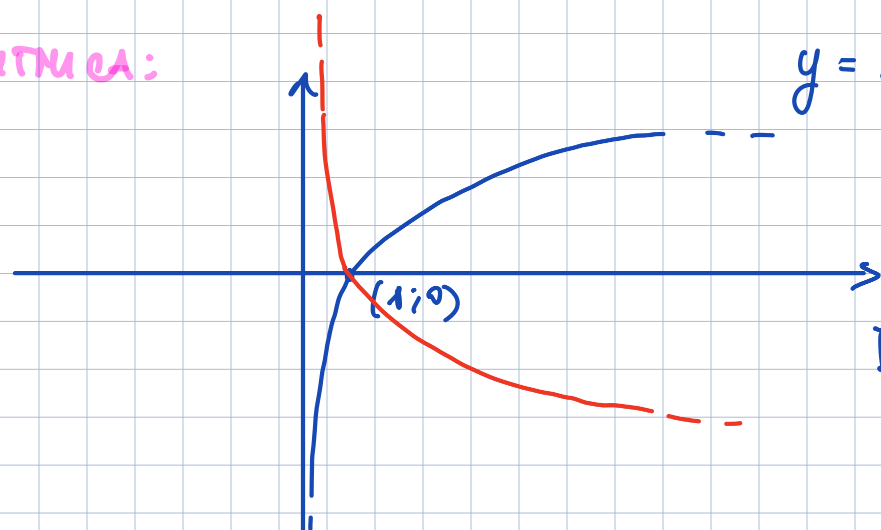
$y = a^x$

$a > 1$

$0 < a < 1$

$D: \mathbb{R}$

FUNZIONE LOGARITMICA:



$y = \log_a x$

$a > 1$

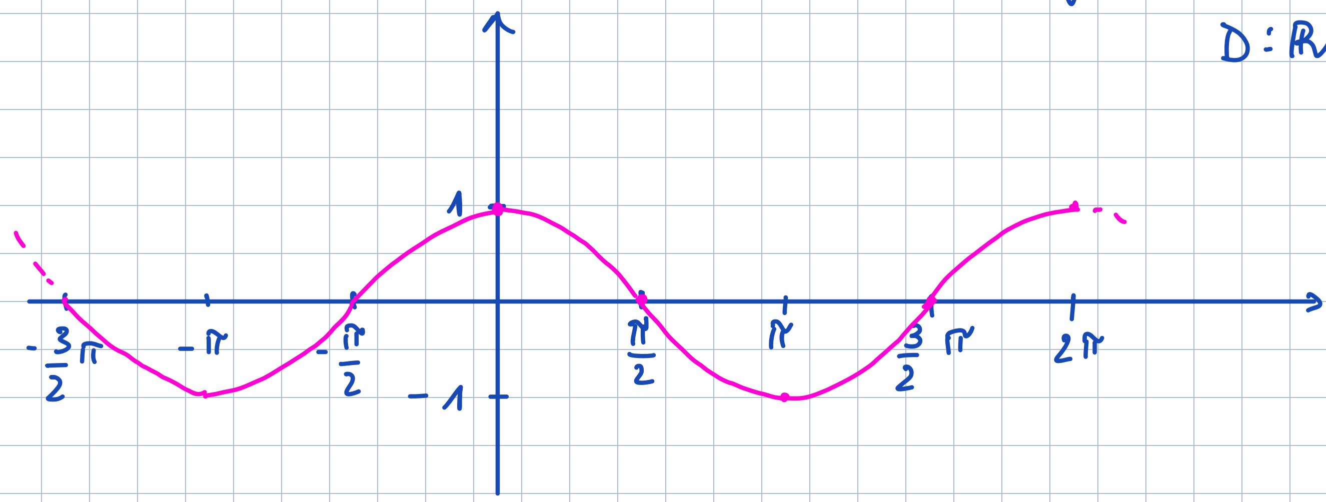
$0 < a < 1$

$D:]0; +\infty[$

FUNZIONE COSENO

$y = \cos x$

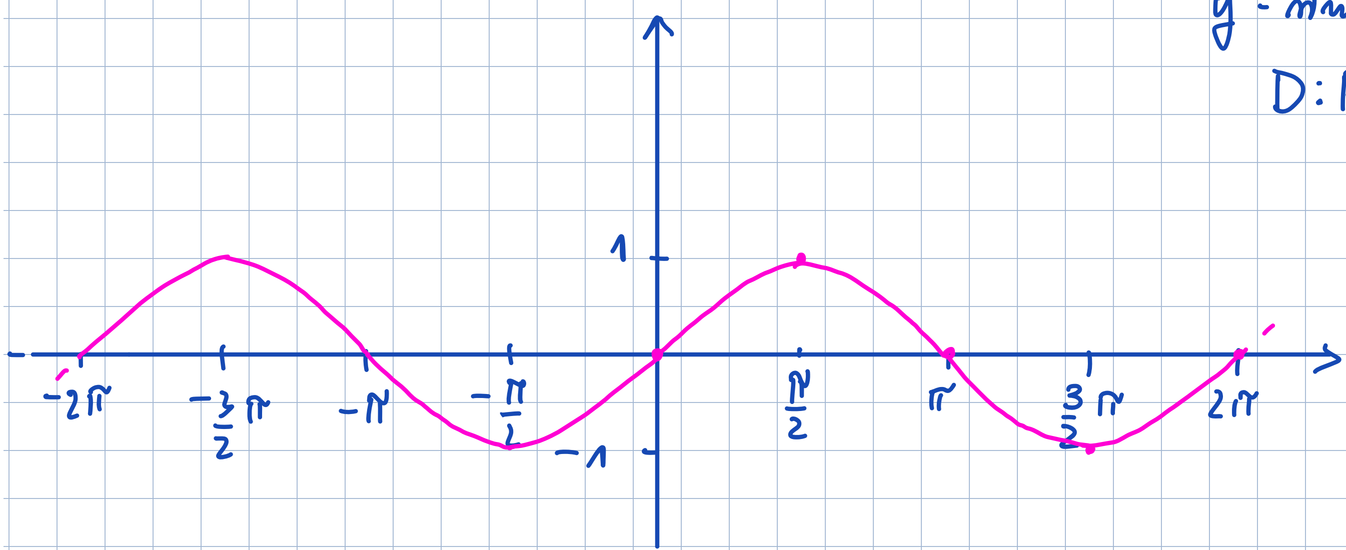
$D: \mathbb{R}$



FUNZIONE SENO

$y = \sin x$

$D: \mathbb{R}$

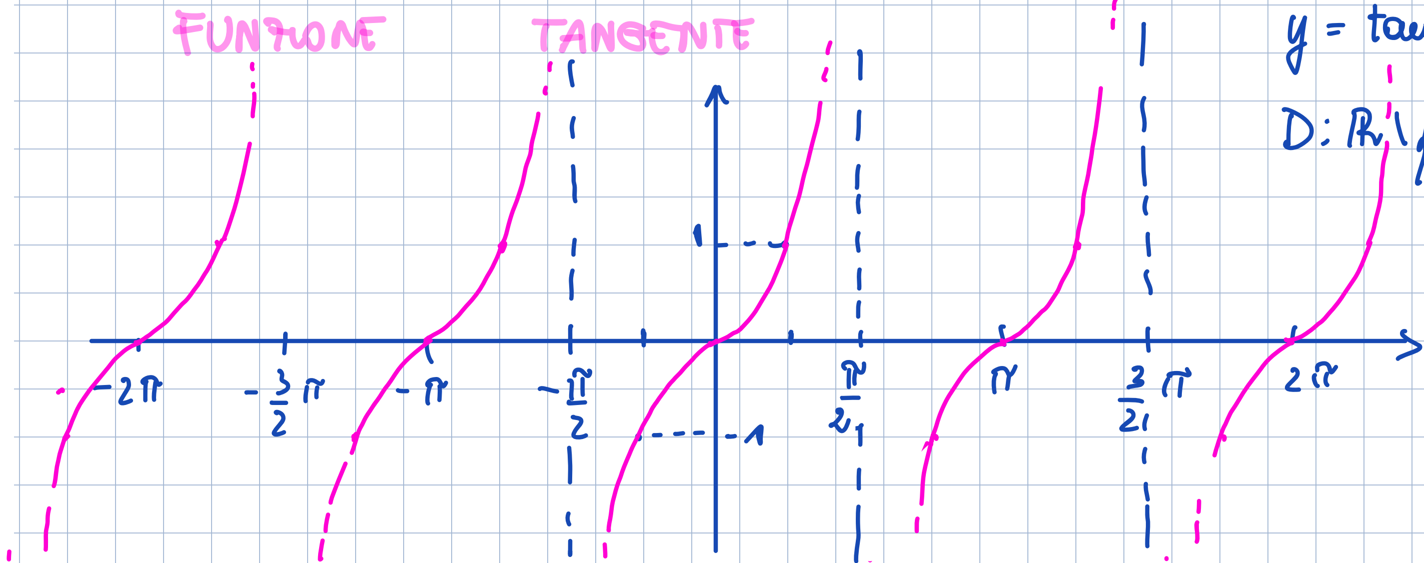


FUNZIONE TANGENTE

$y = \tan x$

$D: \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \right\}$

$k \in \mathbb{Z}$

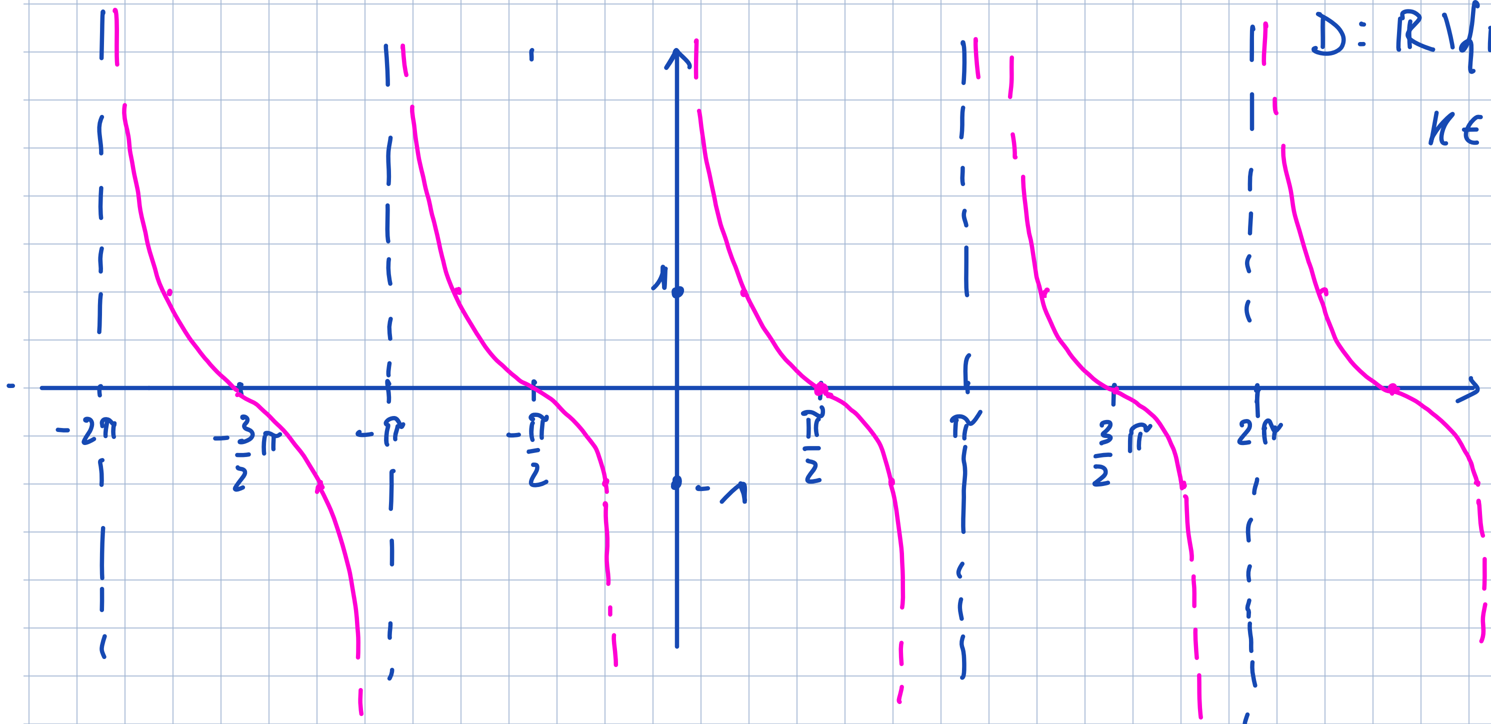


FUNZIONE COTANGENTE

$y = \cot x$

$D: \mathbb{R} \setminus \{\pi + k\pi\}$

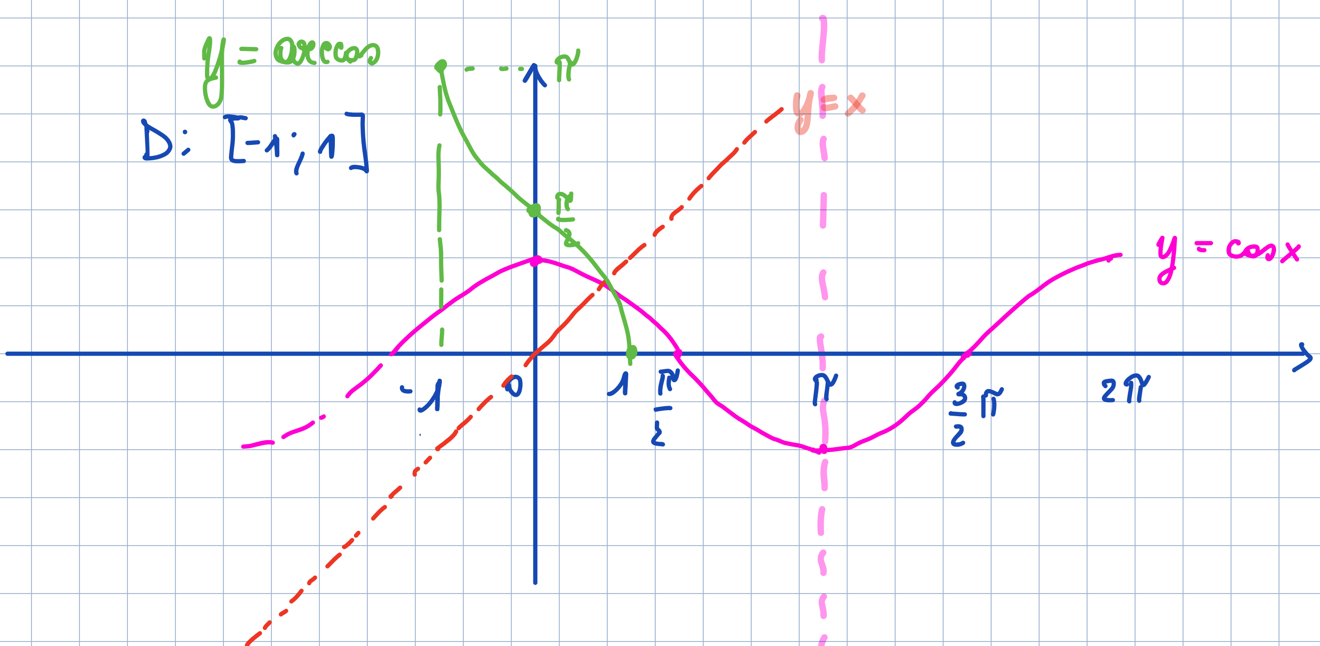
$k \in \mathbb{Z}$



FUNZIONE ARCOCOSENO

$y = \arccos$

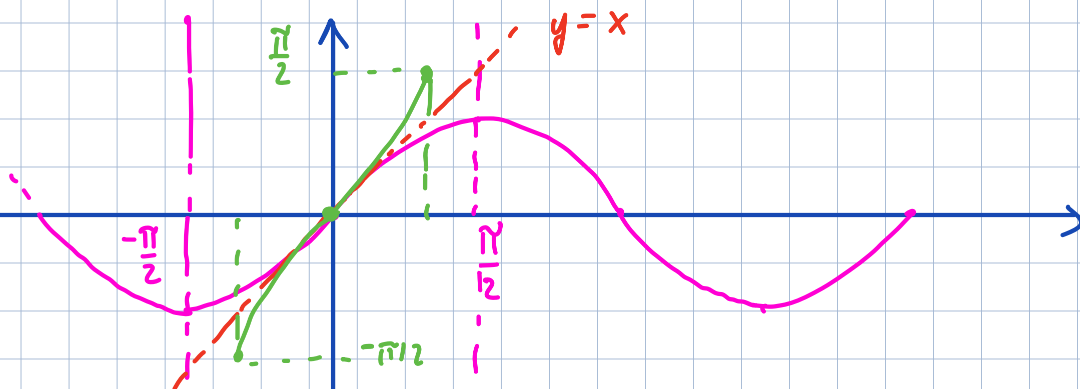
$D: [-1; 1]$



FUNZIONE ARCOSENO

$y = \arcsin x$

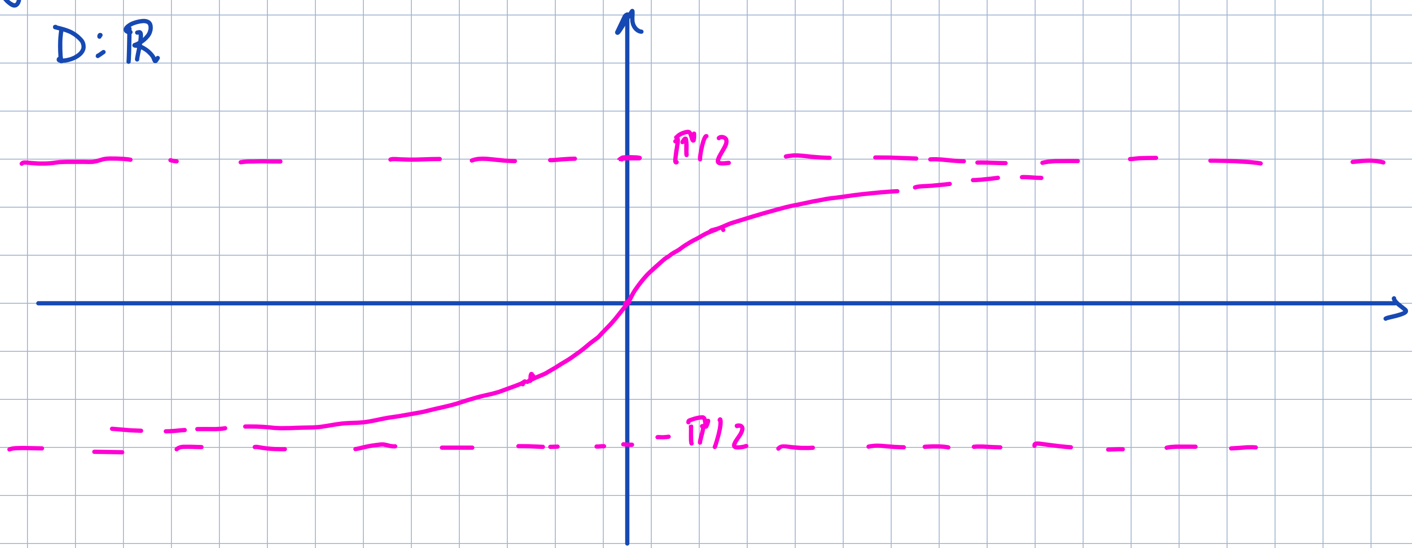
$D: [-1; 1]$



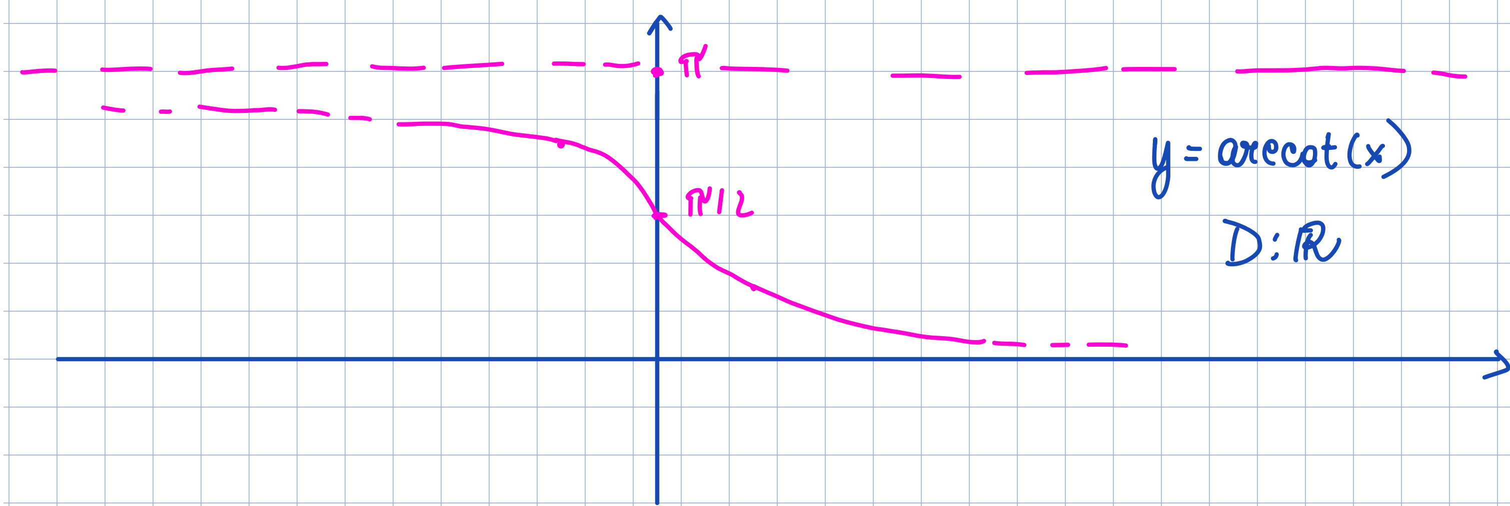
FUNZIONE ARCOTANGENTE

$y = \arctan(x)$

$D: \mathbb{R}$



FUNZIONE ARCOCOTANGENTE



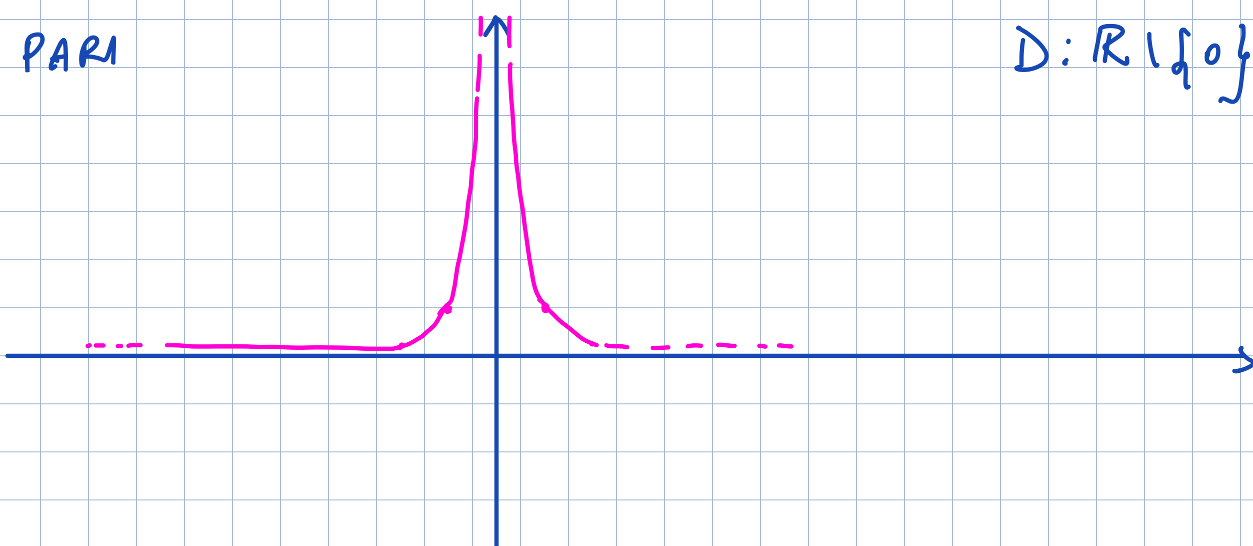
$y = \operatorname{arccot}(x)$

$D: \mathbb{R}$

FUNZIONE POTENZA

$y = \frac{1}{x^n}$ $\quad n \in \mathbb{N} \setminus \{1\}$

1) n PARI $\quad D: \mathbb{R} \setminus \{0\}$



1) n DISPARI $D: \mathbb{R} \setminus \{0\}$

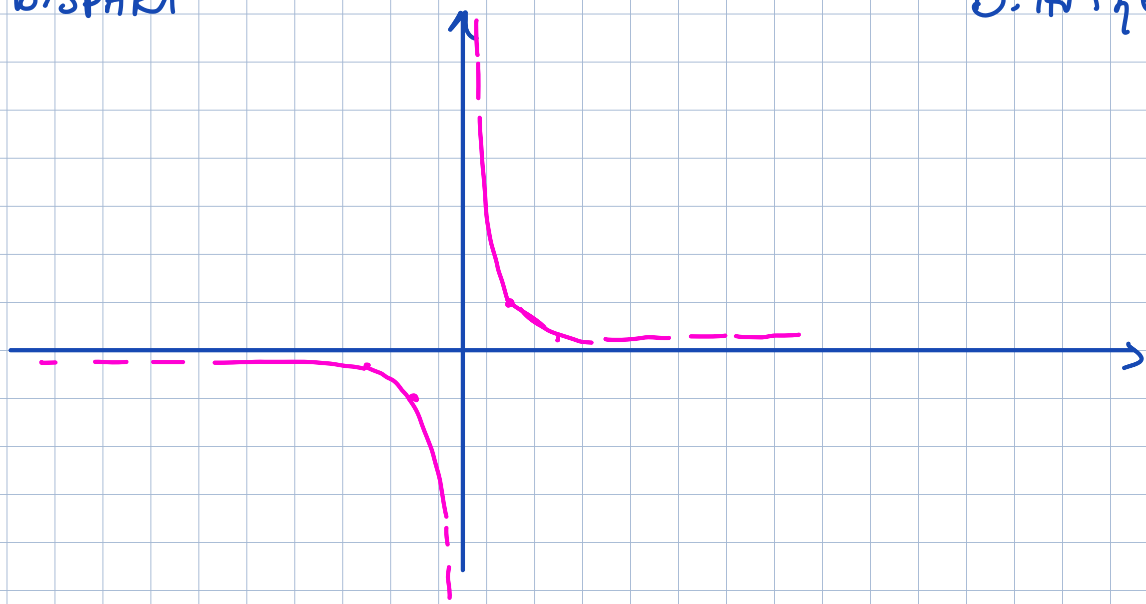


GRAFICO DI UNA FUNZIONE $f: A \longrightarrow B$

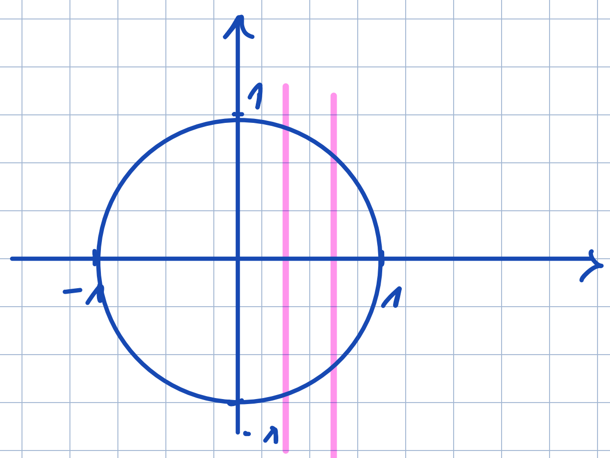
DATA UNA FUNZIONE $f: A \longrightarrow B$ IL SUO GRAFICO È

$$G(f) = \{(x; f(x)) : x \in A\}$$

$G(f)$ È SOTTOINSIEME DI $\mathbb{R}^2$.

CONTROES. L'INSIEME $\{(x; y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} = A$

NON È UN GRAFICO DI ALCUNA FUNZIONE



$$A = \{(x; y) : y = \sqrt{1-x^2} \quad x \in [-1; 1]\}$$

$$\cup$$

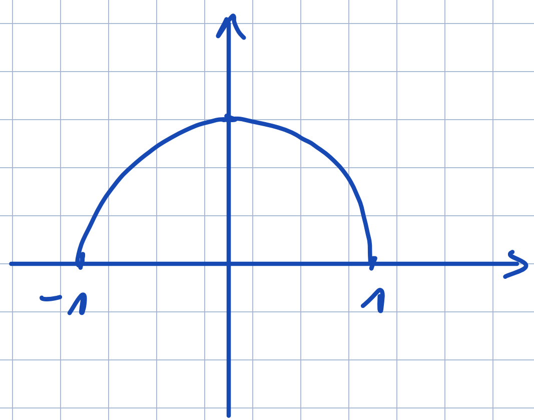
$$\{(x; y) : y = -\sqrt{1-x^2} \quad x \in [-1; 1]\}$$

$A = \{(x; \sqrt{1-x^2}) : x \in [-1;1]\} \cup \{(x; -\sqrt{1-x^2} : x \in [-1;1]\}$

$\exists\ \bar{x} = \frac{\sqrt{3}}{2}$ $\exists\ y_1 = \frac{1}{2}$, $y_2 = -\frac{1}{2}$ t.c. $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\pm\frac{1}{2}\right)^2 = 1$

$f: A \to B$ È UNA FUNZIONE SE $\forall x \in A\ \exists!\ y \in B$ t.c. $f(x) = y$.

OSS: SE CONSIDERO LA SEMICIRCONFERENZA SUPERIORE



$f: [-1;1] \longrightarrow \sqrt{1-x^2}$

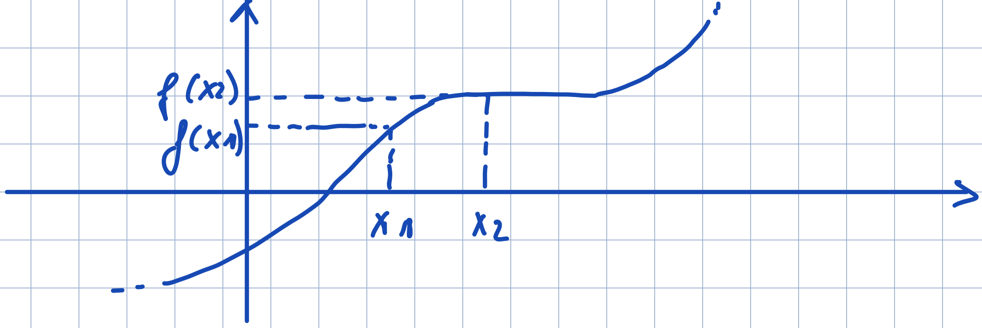
QUESTA RAPPRESENTA IL GRAFICO DI UNA FUNZIONE.

DOMINIO MASSIMALE DI UNA FUNZIONE / CAMPO DI ESISTENZA

$D: \{x \in \mathbb{R} ;\ f(x) \in \mathbb{R}\}$.

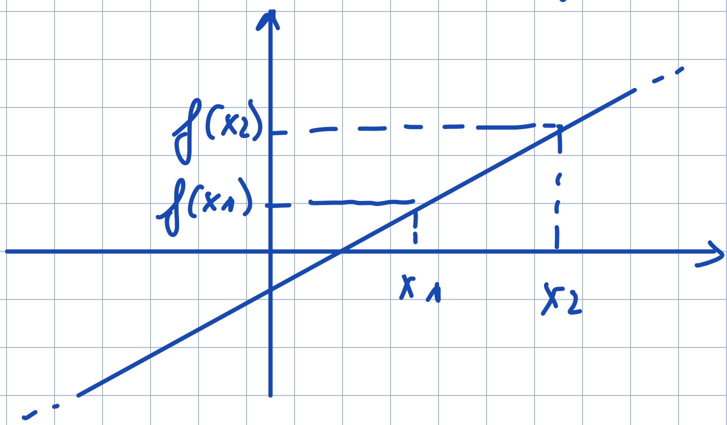
È L'INSIEME DOVE HA SIGNIFICATO $f(x)$.

FUNZIONE DEBOLMENTE CRESCENTE: $f: A \longrightarrow B$ È DEBOLMENTE CRESCENTE: $\forall x_1, x_2 \in A$ $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$



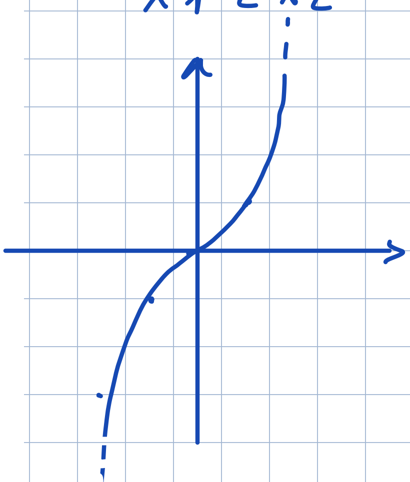
FUNZIONE STRETTAMENTE CRESCENTE: $f: A \to B$

$\forall x_1, x_2 \in A \qquad x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$



ES. $f(x) = x^3$ È STRETTAMENTE CRESCENTE SU $\mathbb{R}$

PRENDO $x_1 < x_2$ con $x_1, x_2 \in \mathbb{R}$



$$x_1 < x_2 \overset{?}{\Rightarrow} f(x_1) < f(x_2)$$

$$x_1^3 < x_2^3$$

$$x_1^3 - x_2^3 < 0$$

$$\underbrace{(x_1 - x_2)}_{<0}(x_1^2 + x_1 \cdot x_2 + x_2^2) < 0$$

FALSO QUADRATO >0

NE SEGUE CHE $x_1^3 < x_2^3 \Leftrightarrow x_1 < x_2$

ES. $f(x) = \begin{cases} 1-x^2 & x \le 0 \\ 1 & x > 0 \end{cases}$ È DEBOLMENTE CRESCENTE $\forall x \in \mathbb{R}$

DEVO PROVARE CHE $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$

1) $x_1 < x_2 \le 0$

$$f(x_2) - f(x_1) = 1 - x_2^2 - (1 - x_1^2) = x_1^2 - x_2^2 = \underbrace{(x_1 - x_2)}_{<0}\underbrace{(x_1 + x_2)}_{<0} > 0$$

$$\left(f(x_2) - f(x_1) > 0 \Longleftrightarrow f(x_1) < f(x_2)\right)$$

c.v.d

2) $x_1 \le 0 < x_2$

$$f(x_2) - f(x_1) = 1 - (1 - x_1^2) = x_1^2 \ge 0$$

c.v.d.

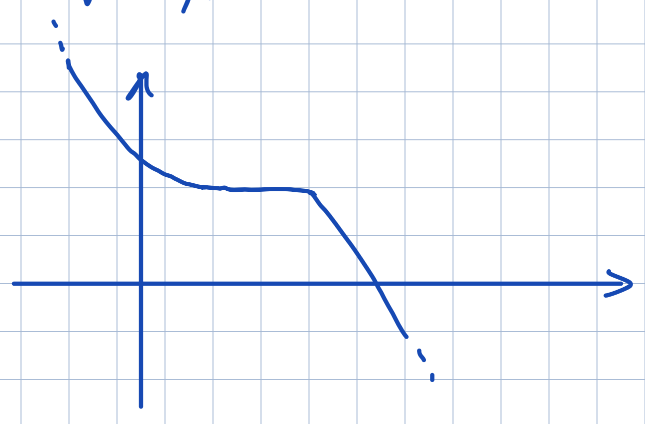
3) $0 < x_1 < x_2$

$$f(x_2) - f(x_1) = 1 - 1 = 0 \ge 0$$

c.v.d.

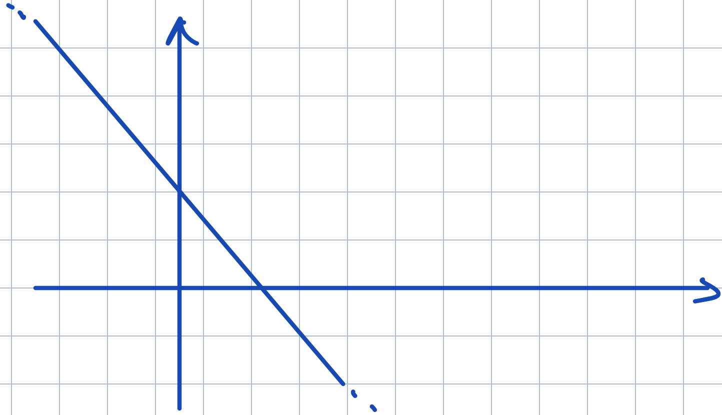
FUNZIONE DEBOLMENTE DECRESCENTE:

$f : A \to B \quad \forall x_1, x_2 \in A \quad x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$



FUNZIONE STRETTAMENTE DECRESCENTE:

$f : A \to B \quad \forall x_1, x_2 \in A \quad x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$



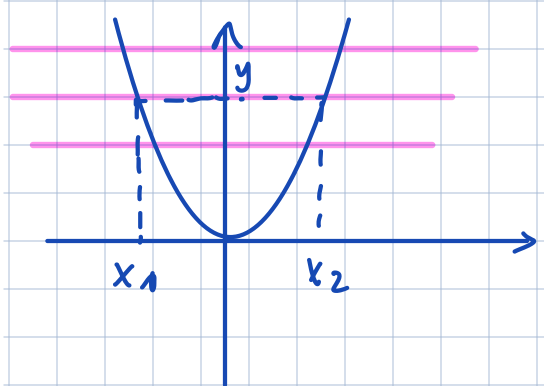
ESEMPIO: $f(x) = x^2$ È STRETTAMENTE DECRESCENTE SU $]-\infty; 0]$. PRENDO $x_1 < x_2$

$$f(x_2) - f(x_1) = x_2^2 - x_1^2 = \underbrace{(x_2 - x_1)}_{>0 \ (\text{es. } -2-(-7))} \underbrace{(x_2 + x_1)}_{<0} < 0$$

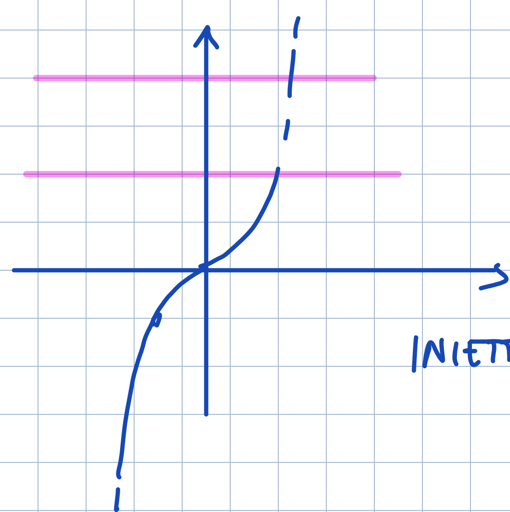
$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ c.v.d.

TEOREMA: $f: A \longrightarrow B$ STRETTAMENTE CRESCENTE (DECRESCENTE) $\Rightarrow f$ È INIETTIVA.

FUNZIONE INIETTIVA: $\forall\, y \in B$ ESISTE AL PIÙ $x \in A$ t.c. $y = f(x)$.



NON INIETTIVA



INIETTIVA

DIM. f STRETTAMENTE CRESCENTE: $\forall\, x_1, x_2 \in A \quad x_1 < x_2$

$$\Downarrow$$

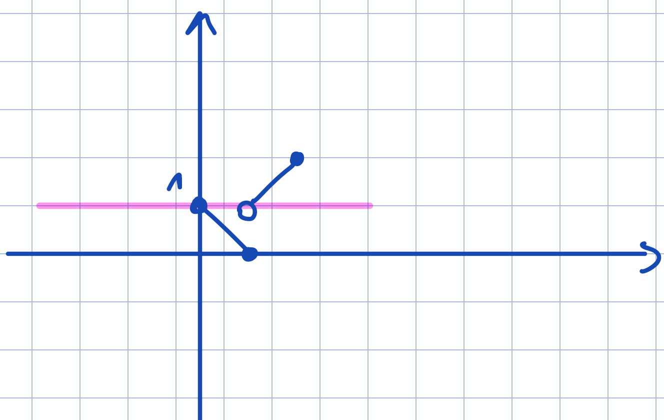
$$f(x_1) < f(x_2)$$

$x_1 \neq x_2 \implies$ ① $x_1 < x_2 \implies f(x_1) < f(x_2)$
② $x_1 > x_2 \implies f(x_1) > f(x_2)$ $\Big\} \implies f(x_1) \neq f(x_2)$

(INIETTIVA: $f(x_1) = f(x_2) \implies x_1 = x_2$) c.v.d.

CONTROESEMPIO: $f: A \to B$ INIETTIVA $\not\Rightarrow$ f STRETTAMENTE CRESCENTE / DEC.

$$f(x) = \begin{cases} 1 - x & 0 \leq x \leq 1 \\ x & 1 < x \leq 2 \end{cases}$$



f È INIETTIVA MA NON STRETTAMENTE CRESCENTE / DECRESCENTE.