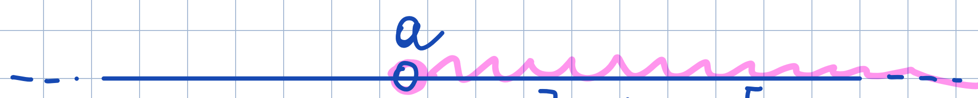
DISEQUAZIONI DI 1° GRADO

$x-a>0 \ (\geq, <, \leq) \qquad a\in\mathbb{R}$

- $\{x\in\mathbb{R} : x-a>0\} \equiv \{x\in\mathbb{R} : x>a\} =]a;+\infty[$

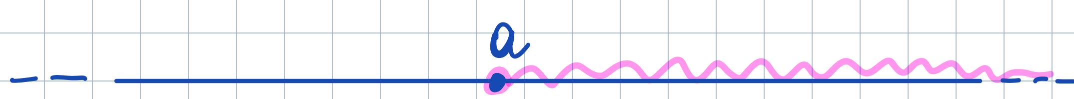
GRAFICAMENTE $\qquad x-a>0$



$]a;+\infty[$

- $\{x\in\mathbb{R} ; x-a\geq 0\} \equiv \{x\in\mathbb{R} : x\geq a\} = [a;+\infty[$

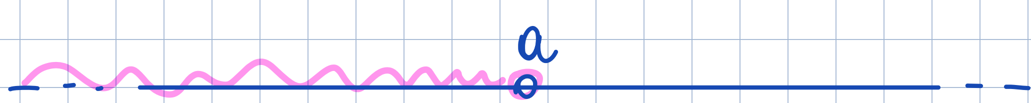
$x-a\geq 0$



$[a;+\infty[$

- $\{x\in\mathbb{R} : x-a<0\} \equiv \{x\in\mathbb{R} ; x<a\} =]-\infty;a[$

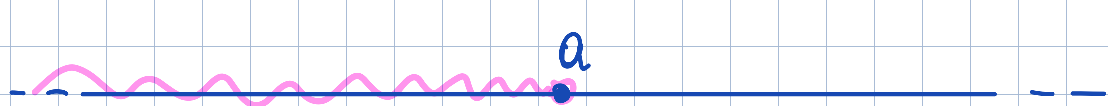
$x-a<0$



$]-\infty;0[$

- $\{x\in\mathbb{R} ; x-a\leq 0\} \equiv \{x\in\mathbb{R} ; x\leq a\} =]-\infty;a]$

$x-a\leq 0$



$]-\infty;a]$

DISEQUAZIONI DI SECONDO GRADO

$$ax^2+bx+c>0 \quad (\geq, \leq, <)$$

$$a, b, c \in \mathbb{R}$$

• SUPPONIAMO $a>0$, NEL CASO IN CUI $a<0$ ANDIAMO A MOLTIPLICARE PER (-1) DA ENTRAMBE I MEMBRI

$$(-1)\ ax^2+bx+c>0\ (-1)$$

$$-ax^2-bx-c<0$$

N.B.

NEL CASO IN CUI $a=0$ ABBIAMO A CHE FARE CON UNA DISEQUAZIONE DI 1° GRADO.

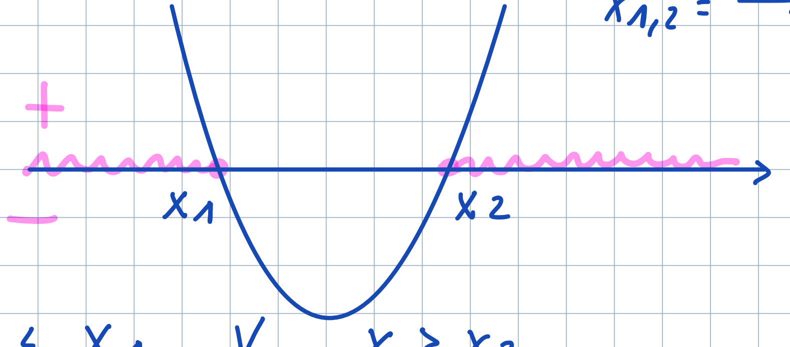
• $ax^2+bx+c \geq 0$

$ax^2+bx+c=0$

$\Delta = b^2-4ac$

$\Delta > 0 \rightarrow$ 2 SOLUZIONI DISTINTE

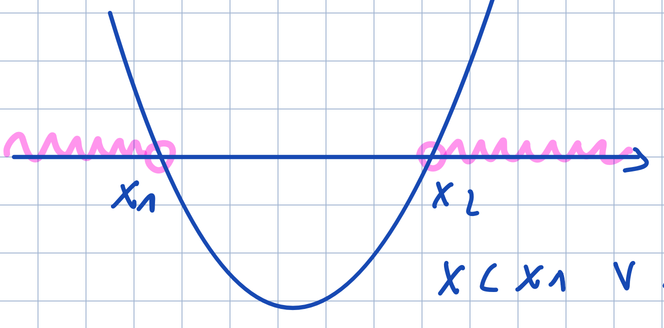
$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$



$x \leq x_1 \vee x \geq x_2$

• $ax^2+bx+c>0$

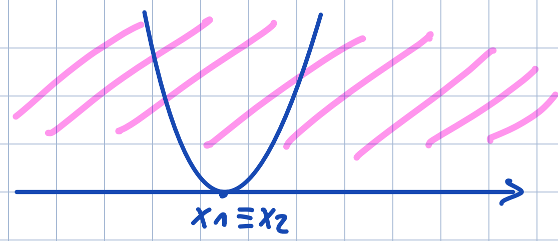
$\Delta > 0 \rightarrow$ 2 SOL. DIST.



$x < x_1 \vee x > x_2$

- $ax^2+bx+c > 0$

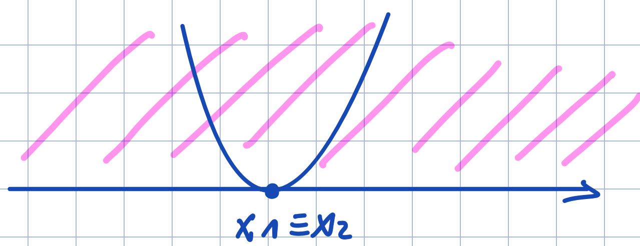
  $\Delta = 0 \longrightarrow 1$ SOL.



$x \neq x_1 \equiv x_2$

- $ax^2+bx+c \geq 0$

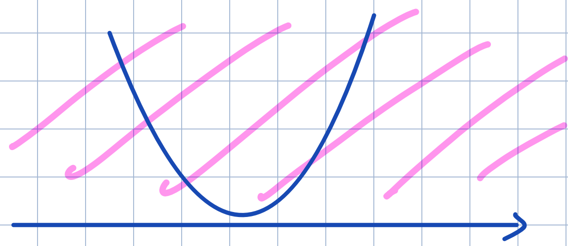
  $\Delta = 0$



$\forall x \in \mathbb{R}$

- $ax^2+bx+c \geq 0$

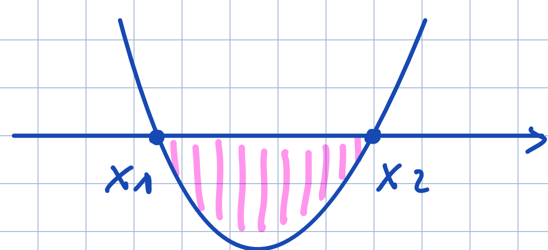
  $\Delta < 0 \longrightarrow \emptyset$ RADICE



$\forall x \in \mathbb{R}$

- $ax^2+bx+c \leq 0$

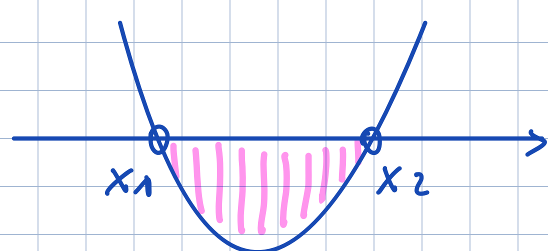
  $\Delta > 0$



$x_1 \leq x \leq x_2$

- $ax^2+bx+c < 0$

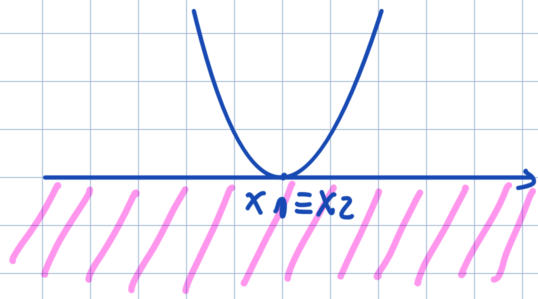
  $\Delta > 0$



$x_1 < x < x_2$

- $ax^2+bx+c<0$

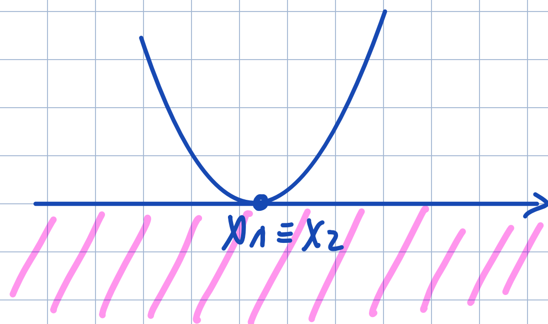
$\Delta=0$



$\nexists\, x \in \mathbb{R}$

- $ax^2+bx+c \leq 0$

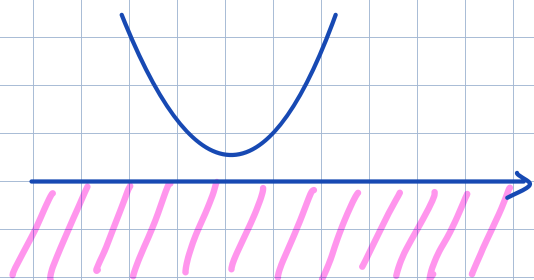
$\Delta=0$



$x = x_1 \equiv x_2$

- $ax^2+bx+c \leq 0$

$\Delta<0$



$\nexists\, x \in \mathbb{R}$

$ax^2+bx+c>0$

$a>0$ HA LE STESSE SOLUZIONI DI

$$x^2+\frac{b}{a}\cdot x+\frac{c}{a}>0$$

CI RIDUCIAMO QUINDI A STUDIARE $x^2+bx+c>0$

$x^2+bx+c<0$

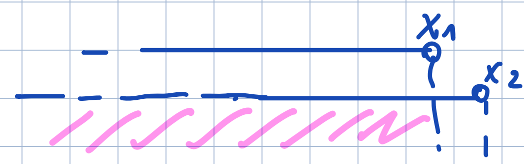
1) RADICI REALI E DISTINTE

$x^2+bx+c=0$ HA 2 RADICI REALI $x_1<x_2$,

ALLORA $\{x : x^2+bx+c>0\} \equiv \{x : (x-x_1)(x-x_2)>0\}$

$\equiv \left(\{x : \underset{x>x_1}{x-x_1>0}\} \cap \{x : \underset{x>x_2}{x-x_2>0}\}\right) \cup \left(\{x : \underset{x<x_1}{x-x_1<0}\} \cap \{x : \underset{x<x_2}{x-x_2<0}\}\right)$

$\equiv \left(]x_1;+\infty[\;\cap\;]x_2;+\infty[\right) \cup \left(]-\infty;x_1[\;\cap\;]-\infty;x_2[\right)$



$]x_2;+\infty[\qquad \cup \qquad]-\infty;x_1[$

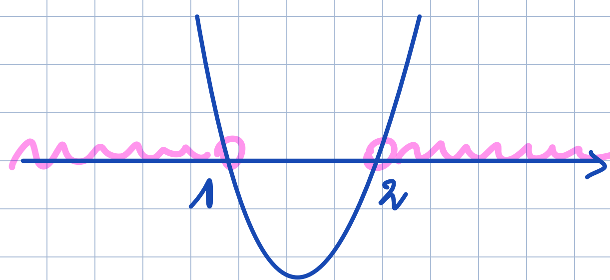
SOLUZIONI: $\quad]-\infty;x_1[\;\cup\;]x_2;+\infty[$

ES. $\quad x^2-3x+2>0$

$x^2-3x+2=0 \qquad x_{1,2} = \dfrac{3\pm1}{2} = \begin{cases} 2 \\ 1 \end{cases}$

$\Delta = 9-8 = 1 > 0$

PRIMO METODO



$x<1 \quad \vee \quad x>2$

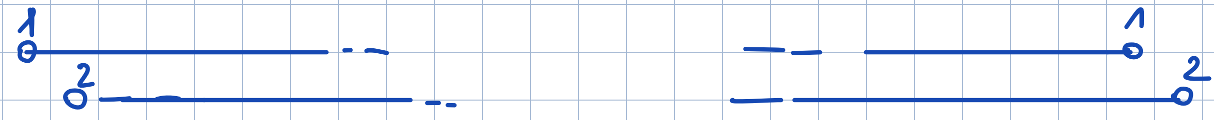
SECONDO METODO $\quad x^2-3x+2 = (x-2)(x-1)$

$(x-2)(x-1)>0$ QUANDO

$((x-1)$ e $(x-2)>0)$ OPPURE $((x-1)$ e $(x-2)<0)$

$(\{x : x>1\} \cap \{x : x>2\}) \cup (\{x : x<1\} \cap \{x : x<2\})$

$(]1;+\infty[\;\cap\;]2;+\infty[) \cup (]-\infty;1[\;\cap\;]-\infty;2[)$



$$]2;+\infty[\quad \cup \quad]-\infty;1[$$

2) RADICI REALI COINCIDENTI $x_1 = x_2 = \bar{x}$

IN QUESTO CASO $x^2+bx+c = (x-\bar{x})^2 > 0 \iff x-\bar{x} \neq 0$

$x \neq \bar{x}$

QUINDI LA SOLUZIONE È $\mathbb{R} \setminus \{\bar{x}\}$

ES. $x^2-2x+1>0 \iff (x-1)^2>0 \iff x \neq 1$

3) RADICI COMPLESSE CONIUGATE $x_1, x_2 \notin \mathbb{R}$

IN QUESTO CASO NEL TRINOMIO x^2+bx+c SI HA CHE

$b^2-4ac<0 \xrightarrow{a=1} b^2-4c<0$

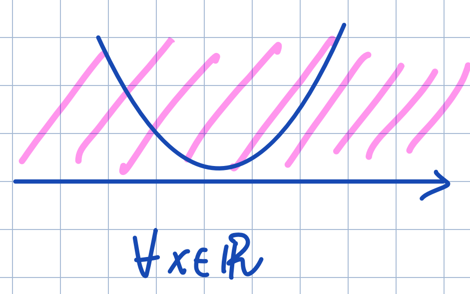
$$x^2+bx+c = \left(x^2+2\cdot\frac{b}{2}x\right)+c$$

$$\left(x^2+2\cdot\frac{b}{2}x+\frac{b^2}{4}\right)+c-\frac{b^2}{4}$$

$$\left(x+\frac{b}{2}\right)^2+c-\frac{b^2}{4} \geq \frac{4c-b^2}{4} > 0 \quad \forall x \in \mathbb{R}$$

ESEMPIO $x^2+2x+5>0$

$\Delta = 4-20<0$



$\forall x \in \mathbb{R}$

$(x^2+2x)+5>0$

$(x^2+2x+1)+5-1>0$

$(x+1)^2+4>0$

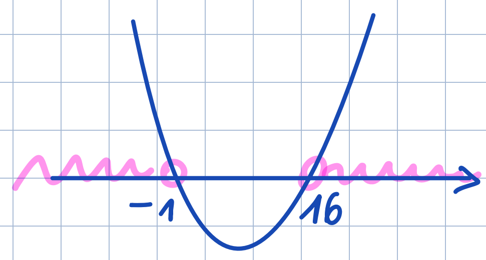
$(x+1)^2>-4 \qquad \forall x \in \mathbb{R}$

ESERCIZIO 2.11:

a) $x^2-15x-16>0$

$\Delta = 225+64 = 289 = 17^2$

$x_{1,2} = \dfrac{15 \pm 17}{2} = \begin{cases} 16 \\ -1 \end{cases}$

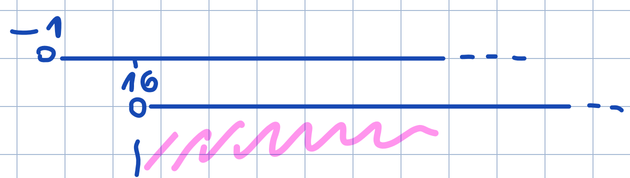


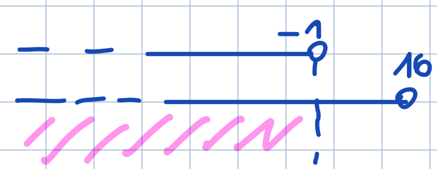
$x<-1 \ \vee \ x>16$

$x^2-15x-16>0 \longrightarrow (x-16)(x+1)>0 \iff$

$\left(\{x : x-16>0\} \cap \{x : x+1>0\}\right) \cup \left(\{x : x-16<0\} \cap \{x : x+1<0\}\right)$

$\left(]16;+\infty[\ \cap \ ]-1;+\infty[\right) \cup \left(]-\infty;16[\ \cap \ ]-\infty;-1[\right)$





$]16;+\infty[\quad \cup \quad]-\infty;-1[$

ES. 2.20:

a) $x^2>8$

b) $x^2-3x<0$

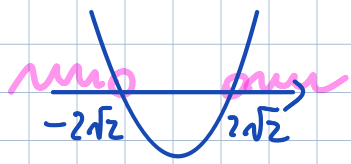
c) $x^2+5x+1 \geq 0$

d) $x^2-3ax+2a^2 \leq 0$

a) $x^2 > 8 \longrightarrow x^2 - 8 > 0$

$\Delta = 0 - 1 \cdot 4(-8) = 32 > 0$

$x_{1,2} = \dfrac{0 \pm \sqrt{32}}{2} = \pm \dfrac{4\sqrt{2}}{2} = \pm 2\sqrt{2}$

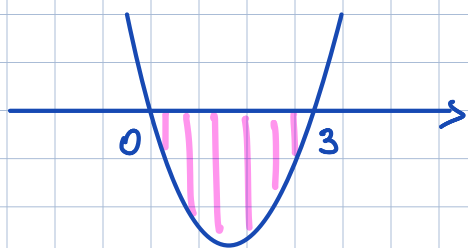


$x < -2\sqrt{2} \quad \vee \quad x > 2\sqrt{2}$

$x^2 = 8 \longrightarrow x = \pm\sqrt{8} \longrightarrow x = \pm 2\sqrt{2}$

b) $x^2 - 3x < 0 \longrightarrow x^2 - 3x = 0 \longrightarrow x(x-3) = 0$

$x = 0 \quad \vee \quad x = 3$

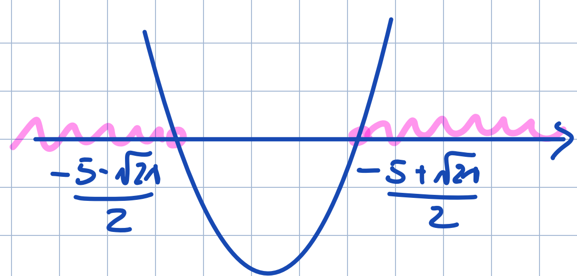


$0 < x < 3$

c) $x^2 + 5x + 1 \geq 0$

$\Delta = 25 - 4 = 21$

$x_{1,2} = \dfrac{-5 \pm \sqrt{21}}{2}$



$x < \dfrac{-5-\sqrt{21}}{2} \quad \vee \quad x > \dfrac{-5+\sqrt{21}}{2}$

d) $x^2 - 3ax + 2a^2 \leq 0$

$\Delta = (-3a)^2 - 4(2a^2) = 9a^2 - 8a^2 = a^2$

$\longrightarrow a^2 < 0$ IMPOSSIBILE

$\longrightarrow a^2 = 0 \quad a = 0$

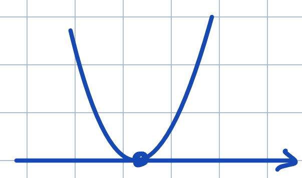
$\longrightarrow a^2 > 0 \quad \forall a \in \mathbb{R} \setminus \{0\}$

$\Delta < 0$ IMPOSSIBILE

$\Delta = 0 \rightarrow a = 0 \rightarrow$ 1 SOLUZIONE

$\Delta > 0 \rightarrow \forall a \in \mathbb{R} \setminus \{0\} \rightarrow$ 2 SOLUZIONI DISTINTE

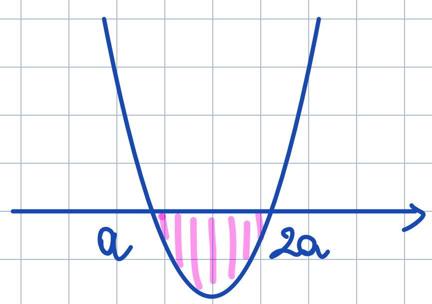
$\Delta = 0 \qquad x^2 \leq 0$



$x = 0$

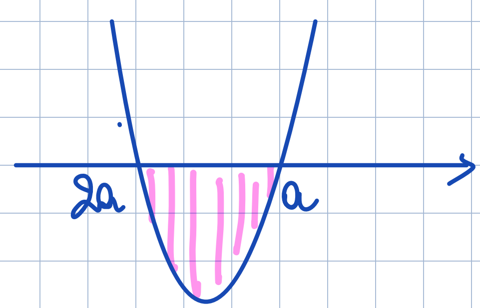
$\Delta > 0 \qquad \Delta = a^2 \qquad x_{1,2} = \dfrac{3a \pm \sqrt{a^2}}{2} = \begin{cases} \dfrac{3a-a}{2} = a \\ \dfrac{3a+a}{2} = 2a \end{cases}$

$a > 0$



$a \leq x \leq 2a$

$a < 0$



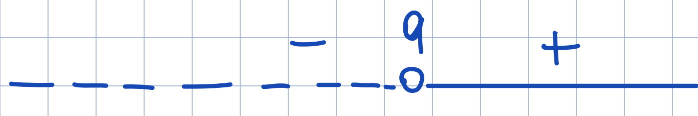
$2a \leq x \leq a$

$$S: \begin{cases} [a; 2a] & a > 0 \\ 0 & a = 0 \\ [2a; a] & a < 0 \end{cases}$$

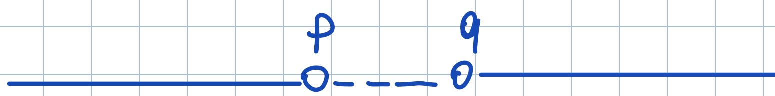
PER RISOLVERE $(x-p)(x-q) > 0$ È SUFFICIENTE STUDIARE $(p < q)$

$x - p > 0 \longrightarrow x > p$

$x - q > 0 \longrightarrow x > q$



$(x-p)(x-q) > 0$



$]-\infty; p[\ \cup \ ]q; +\infty[$

ESEMPIO

PER QUALI $x \in \mathbb{R}$ $\quad (x-1)(x-2)(x-3) \geq 0$?

$x - 1 \geq 0 \longrightarrow x \geq 1$

$x - 2 \geq 0 \longrightarrow x \geq 2$

$x - 3 \geq 0 \longrightarrow x \geq 3$

	1	2	3
−	+	+	+
−	−	+	+
−	−	−	+
−	+	−	+

$1 \leq x \leq 2 \quad \vee \quad x \geq 3$